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MATHEMATICS

P r e p a r a t i o n 10

M o d u l e s 1 – 5



Learning
Technologies
Branch

Teacher's Guide

Alberta
LEARNING

1 2 3 4 5 1 2 3 4 5 6 7

Mathematics Preparation 10

TEACHER'S GUIDE



NOTE: This Mathematics Preparation 10 Teacher's Guide contains the Final Test and answers to the Final Test and to the teacher-assessed assignments; therefore, teachers should at all times keep it secure against unauthorized student access. The Final Test should be accessible to students only in supervised situations.

Mathematics Preparation 10
Modules 1–5
Teacher's Guide
Learning Technologies Branch
ISBN 0-7741-1741-9

This document is intended for

Students	
Teachers	✓
Administrators	
Parents	
General Public	
Other	



You may find the following Internet sites useful:

- Alberta Learning, <http://ednet.edc.gov.ab.ca>
- Learning Technologies Branch, <http://www.learning.gov.ab.ca/ltb>
- Learning Resources Distributing Centre, <http://www.lrdc.gov.ab.ca/lrdc>

The use of the Internet is optional. Exploring the electronic information superhighway can be educational and entertaining. However, be aware that these computer networks are not censored. Students may unintentionally or purposely find articles on the Internet that may be offensive or inappropriate. As well, the sources of information are not always cited and the content may not be accurate. Therefore, students may wish to confirm facts with a second source.

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Teachers

Register with the Learning Technologies Branch

The Learning Technologies Branch is dedicated to upgrading and continually improving your Teacher's Guide so that the guide accurately reflects any necessary revisions we have had to make in the Student Module Booklet, Assignment Booklet, or the sample Final Test. The types of revisions that will be made are those that make the course more accurate, more current, or more effective.

LTB will send you the **latest enhancements** or **minor upgrades** for your Teacher's Guide if you register with us. To do so, please fill out the registration portion of this page and send this page to: Learning Technologies Branch, Box 4000, Barrhead, Alberta T7N 1P4, Attention: Instructional Design and Development.

LTB Teacher's Guide Registration Form

First Name

Surname

School Name

School Phone Number

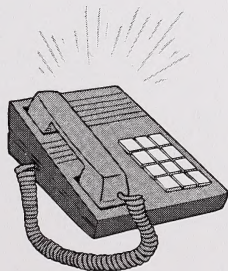
School Address

City

Postal Code

Course Title

Approximate Date of Purchase



You can help ensure that distance learning courseware is of top quality by letting us know of areas that need to be adjusted. Call the Learning Technologies Branch—free of charge by dialling first 310-0000, then 674-5350—and ask for the Editing Unit. Also, a teacher questionnaire has been included at the back of most Teacher's Guides. Please take a moment to fill it out and send it in.

We look forward to hearing from you!

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Introduction

A survey of these course materials will confirm that this learning package has been specially designed for many kinds of teachers working in a variety of situations.

In Which Category Do You Fit?

- ☐ Small Schools Teacher
 - ☐ inexperienced
 - ☐ experienced, but in other subject areas
 - ☐ experienced in teaching Mathematics Preparation 10, but wanting to try a different approach
- ☐ Distance Learning Teacher
 - ☐ travelling to schools within the region
 - ☐ using facsimile and teleconferences to teach students within the region
- ☐ Large Schools Teacher
 - ☐ inexperienced
 - ☐ experienced in teaching Mathematics Preparation 10, but wanting to try a different approach



Because these materials have been created by experienced classroom teachers and distance learning specialists, they have many advantages for students and teachers, regardless of the situation.

Advantages for Students

Materials

- incorporate a strong learner-centred philosophy
- promote such qualities in the learner as autonomy, independence, and flexibility
- are developed through media that suit the needs and circumstances of the learner
- reflect the experiential background of Alberta students
- provide opportunities by overcoming barriers that result from geographical location
- promote individualized learning, allowing learners to work at their own pace

Advantages for Teachers

Materials

- allow teachers maximum teaching time and minimize preparation time
- include different routes through the materials to suit different learners
- incorporate a wide range of teaching strategies, in particular those using independent and individual learning
- deliver curriculum designed by education specialists that reflects the Alberta Learning Program of Studies with an emphasis on Canadian content
- provide learning materials that are upwardly compatible with advanced educational technology

Does this learning package sound like something you could use?

This Teacher's Guide begins with an overview of the current Alberta Learning Program of Studies for Mathematics Preparation 10. This summary is included for inexperienced teachers or those teachers who have found themselves teaching Mathematics Preparation 10 when their training is in other subject areas. This brief description is not meant to replace the Alberta Learning Program of Studies, but rather to help teachers confirm the highlights of the program.

Other parts of this introduction have also been included to help teachers become familiar with this learning package and determine how they might want to use it in their classrooms.

Beyond the introduction, the guide itself contains answers, models, explanations, and other tips generated by the teachers who authored this course.

The Student Module Booklets, Assignment Booklets, and TGs are the products of experienced classroom teachers and distance learning specialists. It is the hope of these teachers that their experience can be shared with those who want to take advantage of it.



Overview of the Program of Studies

The world is changing, workplaces are changing and the mathematics students need to successfully meet the challenge of their studies. The new mathematics curriculum outlines the skills and knowledge students will need to meet these challenges and offers new approaches to integrate technologies into mathematics studies.

The mathematics program focuses study in four key strands:

- Number concepts and operations
- Patterns and relations
- Shape and space
- Statistics and probability

Technology is integrated throughout the new curriculum, including the use of graphing calculators, computer spreadsheets, and so on.

Students must choose the high school mathematics program that best suits their mathematical strengths, their interests, their future educational plans, and their potential career choices.

Applied Mathematics 10–20–30

Applied mathematics emphasizes the application of mathematics and the use of numerical and geometric approaches to solve problems. These courses may be used as entrance requirements to a number of post-secondary programs whose main focus is not mathematics. Students who have passed Grade 9 mathematics should experience success in Applied Mathematics 10.

Pure Mathematics 10–20–30

Pure mathematics emphasizes mathematical theory and the use of algebra and graphing to solve problems. Pure mathematics is required for post-secondary programs such as engineering, physics, and commerce. Students should have a mark of at least 60% in Grade 9 mathematics to ensure success in Pure Mathematics 10.

Mathematics 31

Mathematics 31 is an introductory calculus course. Pure Mathematics 30 is a pre- or corequisite for Mathematics 31.

Mathematics 14–24

These courses will give students high school mathematics credit requirements. These courses can be taken by students who did not pass Grade 9 mathematics.

Mathematics Preparation 10

Students who have had difficulty in mathematics, but wish to take either Applied Mathematics 10 or Pure Mathematics 10 may complete this course. It reviews key junior high mathematics concepts in preparation for the Grade 10 courses.

Bridging Courses

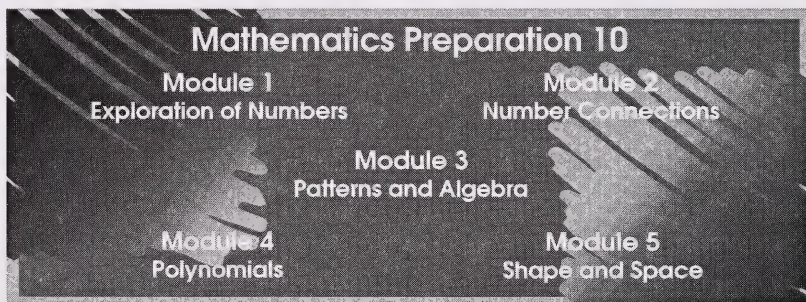
Students may transfer between the Applied and the Pure sequences by completing bridging courses that cover concepts that may not have been taught in the sequence from which they are transferring.

Check the Alberta Learning website for detailed curriculum information at

<http://www.learning.gov.ab.ca/studentprograms/>

Overview of Mathematics Preparation 10

Mathematics Preparation 10 has five modules. Each module is worth one credit. Students may complete one, two, three, four, or five modules. However, most students will take this course for three or five credits. Teachers should ensure that students who are taking this course for fewer than five credits have previously mastered the material in the modules they will not be completing.



The distance learning materials have been designed to clearly indicate the expected student outcomes specified in the program of studies. Concepts are carefully developed and learning aids are frequently used to help students visualize the concepts. The various mathematical processes identified in the program of studies are stressed. The problem-solving process is described in the beginning of each module and many problem-solving strategies are demonstrated in the Appendix of each module. Connections are made with the everyday world and with previously developed skills and knowledge. There is a balance of estimation and mental math, paper-and-pencil exercises, and the appropriate use of calculators and computers. Students are given opportunities to communicate and reason mathematically.

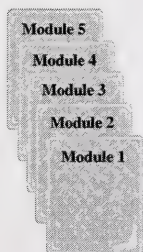
Every attempt has been made to give students a variety of activities and to make mathematics real and interesting. Learning aids and technology are integrated to cater to learning-style preferences of individual students.

Structure of the Learning Package

Basic Design

This learning package involves many other components in addition to the Teacher's Guide. A survey of the components will show you that the course is divided into clumps of learning called modules. For each module there are two print components: a Student Module Booklet and an Assignment Booklet.

Student Module Booklets



Student Module Booklets contain guided activities that instruct students in a relevant, realistic setting.

These booklets have been specially designed to promote such qualities in the learner as autonomy, independence, and flexibility. Writers have incorporated such teaching strategies as working from the concrete to the abstract, linking the old to the new, getting students actively involved, and using advance, intermediate, and post organizers. Many other techniques are used to cater to individual learning styles and preferences. The materials have been designed to include a variety of pathways and options because they are intended for a broad range of use within and beyond Alberta.

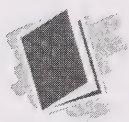
Contents
Overview Evaluation
Section 1 Activity 1 Activity 2 etc.
Section 2 Activity 1 Activity 2 etc.
Module Summary
Appendix

The structure of the Student Module Booklets follows a systematic design. Each booklet begins with a detailed table of contents that shows the students all of the main steps; this page acts as an organizer for students. The Module Overview introduces the module topic or theme and includes a graphic representation to help visual learners and poor readers. This introduction also includes an evaluation statement, so students are informed of the weightings of each assignment.

The body of the Student Module Booklet is made up of two or more closely related sections. Each section contains student activities that develop skills and knowledge centred around a theme. The activities may include print, audio, video, computer, or Internet involvement. Computer and Internet activities are optional. At times, the student and the teacher are allowed to choose the activity that best suits the student's needs and interests. This flexibility caters to each student's personal situation.

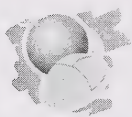
Following the last section is a modular summary that focuses on the skills and strategies that the student has learned. The Student Module Booklet ends with an Appendix that includes a Glossary and Suggested Answers for the self-assessment work.

Assignment Booklets



Accompanying each Student Module Booklet are the Assignment Booklets. The activities in the Assignment Booklets can be used for both formative and summative assessments. Students should complete these assignments when they have thoroughly reviewed the other module materials. Students should submit their work for assessment as soon as they have completed each Assignment Booklet. The Assignment Booklets have been designed for both in-classroom use and for distance learning.

Media



INTERNET

Internet references are found throughout the course; exploring those references, however, is optional for students. In most cases, students are given suggested topics that they may explore through the Internet's search engines. In rare cases, students might be given a specific uniform resource locator (URL), but they should be aware that these addresses are subject to change.

Updated information about the Learning Technologies Branch and this course and others can be found by starting at the Learning Technologies Branch's home page at

<http://www.learning.gov.ab.ca/ltb/>

or by going straight to the LTB Courses page at

<http://www.learning.gov.ab.ca/ltb/400/courses.html>

Materials, Media, and Equipment

Mandatory Components

Equipment (Hardware)	Media	Materials
• scientific calculator • geometry set		• Teacher's Guide for Mathematics Preparation 10 • one complete set of Student Module Booklets (5) and Assignment Booklets (12) for each student • There is a Final Test.

Optional Components

Equipment (Hardware)	Media	Materials
• computer	• the Internet • spreadsheet software (such as <i>ClarisWorks</i>™ and <i>Excel</i>®)	

Using This Learning Package in the Classroom

Conventional Classroom

Whether your classroom has desks in rows or tables in small groups, you may be most comfortable with a learning system that you can use with all your students in a paced style. In other words, you may want a package that will suit all of your students, so they can move through the materials as one group or several small groups. Because these materials contain different routes or pathways within each module, they can address various learning styles and preferences. The materials also include many choices within the activities to cater to different thinking levels and ability levels. Because of their versatility and flexibility, these materials can easily suit a conventional classroom.

Open Learning Classroom

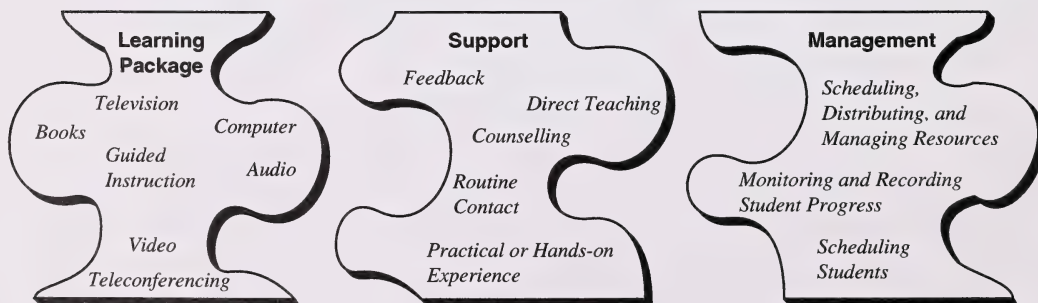
Open learning is the concept of opening up opportunities by overcoming barriers of time, pace, and place by giving the learners a package specially designed to enable them to learn on their own for at least some of the time.

Such a concept is not new. Many teachers can recite attempts to establish an individualized learning system as they recognized the importance of trying to personalize courseware to meet each individual's needs. But these past efforts often failed because of lack of time and lack of quality materials that conformed to Alberta specifications.

Owing to advanced educational technology and improved Alberta-specific learning packages, a student-centred approach is now possible. Improved technology now allows us to provide support to learners individually, regardless of their pace or location. A teacher cannot be in twenty-eight places at one time offering guidance. Media and a well-designed learning package, however, can satisfy individual needs. Technology can also help provide an effective management system needed to track the students as they progress independently through the materials.

The key to a successful open learning system depends on three vital elements: a learning package specially designed to enable students to learn effectively on their own for at least some of the time; various kinds of learner support; and a management system and style that ensures that the open learning system runs smoothly.

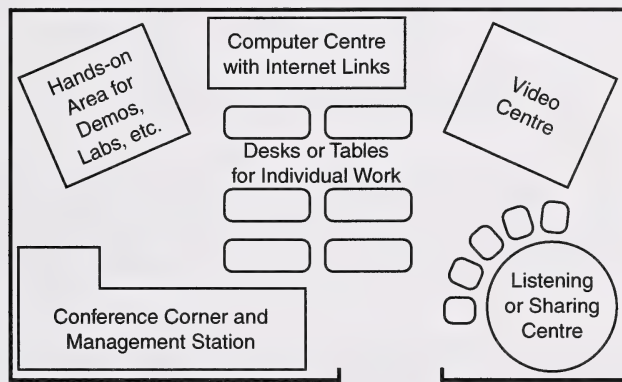
The Key to a Successful Open Learning System



Learning Package

The specially designed learning package needed for a successful open learning system has been developed for you. The objectives teach current Alberta specifications using strategies designed for individualized instruction. As the teacher, you need to be sure to have all of the components in the learning package available to students as needed.

If you are able to acquire media and appropriate hardware to meet your class needs, media centres can be established.



You may not have the luxury to have enough hardware to set up a permanent video or computer centre in your classroom. In that case, students should be encouraged to plan ahead. Perhaps every three to five days they should preview their materials and project when they would need a certain piece of media. This would allow you to group students, if necessary, or reserve media as required.

Support

Support is definitely a key element for successful learning, and when you're planning an individualized, non-paced program, you need to carefully plan when and how support will be given.

The materials contain a form of consistent support by providing immediate feedback for activities included in the Student Module Booklet. Students have solutions, models, explanations, and guides included in the Appendix of every booklet. These aids are included so students can receive immediate feedback to clarify and reinforce their basic understanding before they move on to higher levels of thinking.

As the teacher, you may need to offer more support and personal guidance to those students having difficulty. The activities include choices and pathways. If a student is struggling, you may need to encourage that student to work on all of the choices rather than on only one. This would provide additional instruction and practice in a variety of ways.

You may also have to reinforce the need for students to do the sectional activities carefully and thoroughly before attempting the assignments in the Assignment Booklets.

Another form of support is routine contact with each individual. This contact might be achieved with a biweekly conference scheduled by you; or, as students reach a certain point (e.g., after each section is completed), they may be directed to come to the conference area.

Special counselling may be needed to help students through difficult stages. Praise and encouragement are important motivators, particularly for those students who are not used to working independently.

Direct teaching may be needed and scheduled at certain points in the program. This teaching might involve small groups or a large group. It might be used to take advantage of something timely (e.g., election, eclipse), something prescheduled like the demonstration of a process, or something involving students in a hands-on, practical experience.

Support at a distance might include tutoring by phone, teleconferencing, faxing, or planned visits. These contacts are the lifeline between learners and distance education teachers, so a warm dialogue is essential.

Management

Good management of an open learning system is essential to the success of the program. Leadership and promotion of the system are essential. The following areas also need action to ensure that the system runs smoothly:

- **Scheduling, Distributing, and Managing Resources**—As discussed earlier, in order to do these tasks efficiently you may need to develop media centres or a system for students to reserve the necessary resources.
- **Scheduling Students**—Students and teachers should work together to establish goals, course completion timelines, and daily timelines. Although students may want to study for long periods of time (e.g., all morning), teachers should discourage this practice. Concentration, retention, and motivation are improved by taking scheduled breaks.
- **Monitoring Student Progress**—You will need to record the date that each student completes each module and the Final Test. Your data might also include the projected date of completion if you are using a student-contract approach.



Sample of a Student Progress Chart

Mathematics Preparation 10		Module 1	Module 2	Module 3	Module 4	Module 5	Final Test
<i>Billy Adams</i>	P						
	A						
<i>Louise Despins</i>	P						
	A						
<i>Violet Kjaissian</i>	P						
	A						
P = Projected Completion Date A = Actual Completion Date							

The student could keep a personal log as well. Such tracking of data could be stored easily on a computer.

- **Recording Student Assessments**—You will need to record the marks awarded to each student for work completed in each Assignment Booklet. The marks from these Assignment Booklets will contribute to a portion of the student's final mark. Other criteria may also be added (a special project, effort, attitude, etc.). Whatever the criteria, they should be made clear to all students at the beginning.

Note: In Mathematics Preparation 10, a student may complete any or all of the modules.

Sample of a Student Assessment Chart

Mathematics Preparation 10	Module 1	Module 2	Module 3	Module 4	Module 5	Year's Average	Other	Final Test	Final Mark
<i>Billy Adams</i>									
<i>Louise Despins</i>									
<i>Violet Kjaissian</i>									

Letter grading could easily be substituted.

- **Recording Effectiveness of System**—Keep ongoing records of how the system is working. This data will help you in future planning.

Sample of a System Assessment Chart

Mathematics Preparation 10: Module 1			
Date	Student Module Booklet	Assignment Booklets	Resources/Media

The Role of the Teacher in an Open Learning Classroom

The teachers in a conventional classroom spend a lot of time talking to large groups of learners. The situation in open learning requires a different emphasis. Teachers will probably meet learners individually or in very small groups.

With this approach, it is necessary to move beyond the idea of a passive learner depending largely on a continually supportive teacher. The teacher must aim to build the student's confidence, to stimulate the learner into self-reliance, and to guide the learner to take advantage of routes that are most meaningful and applicable to the learner.

These materials are student centred, not teacher centred. The teacher needs to facilitate learning by providing general support to the learner.

Assessment

Assessment is important to the development of every learner. In order to identify strengths and weaknesses, both students and teachers need to gather and analyse information and make decisions.

These specially designed learning packages contain many kinds of informal and formal assessment.

Informal Assessment

Observation

In the classroom, the teacher has the opportunity to see each student perform every day and to become aware of the level and nature of each student's performance.

Observations are more useful if they are recorded in an organized system. The following list of questions is a sample of types of observations and how they can be collected.

Observation Checklist

	B. Adams	L. Despins	V. Klaissian	H. Smith	K. Dalley
1. Does the student approach the work in a positive manner?					
2. Is the student struggling with the reading level?					
3. Does the student make good use of time?					
4. Does the student apply an appropriate study method?					
5. Can the student use references effectively?					

Observation may suggest a need for an individual interview with a student.

Individual Conferences

Individual conferences may be paced (scheduled) by the calendar, designated at certain points in the module, or they may be set up only as needed or requested.

During these conferences, the teacher can determine the student's progress and can assess the student's attitudes towards the subject, program, school, and self, as well as the student's relationship with other students. With guided questions, the teacher can encourage oral self-assessment; the student can discuss personal strengths or weaknesses in regard to the particular section, module, or subject area.

Self-Appraisal

Self-appraisal helps students recognize their own strengths and weaknesses. Through activities that require self-assessment, students also gain immediate feedback and clarification at early stages in the learning process. Teachers need to promote a responsible attitude towards these self-assessment activities. Becoming effective self-assessors is a crucial part of becoming autonomous learners. By instructing, motivating, providing positive reinforcement, and systematically supervising, learning facilitators will help students develop a positive attitude towards their own progress.

For variation, students may be paired and peer-assessing may become part of the system. The teacher may decide to have the student self-assess some of the activities, to have a peer assess other activities, and to become directly involved in assessing the remainder of the activities.

When the modular activities have been assessed, students should be directed to make corrections. This task should be made clear to students right from the beginning. It is important to establish the correct association between the question and the response to clarify understanding, aid retention, and be of use for study purposes.

Many of the activities include choices for students. If students are having difficulty, more practice may be warranted, and students may need to be encouraged to select more of the choices.

Each section within a Student Module Booklet includes additional types of activities called Extra Help and Enrichment. Students are expected to be involved in the decision as to which pathway best suits their needs. They may decide to do both.

Self-appraisal techniques can also be introduced at the individual conferences. Such questions as the following might be included:

- What steps are you taking to improve your understanding of this topic?
- What method of study do you use most?
- How do you organize the material to help you remember it?
- What steps do you follow when you complete an assignment?
- What could you do to become an even better reader?
- Do you have trouble following directions?
- Did you enjoy this module?

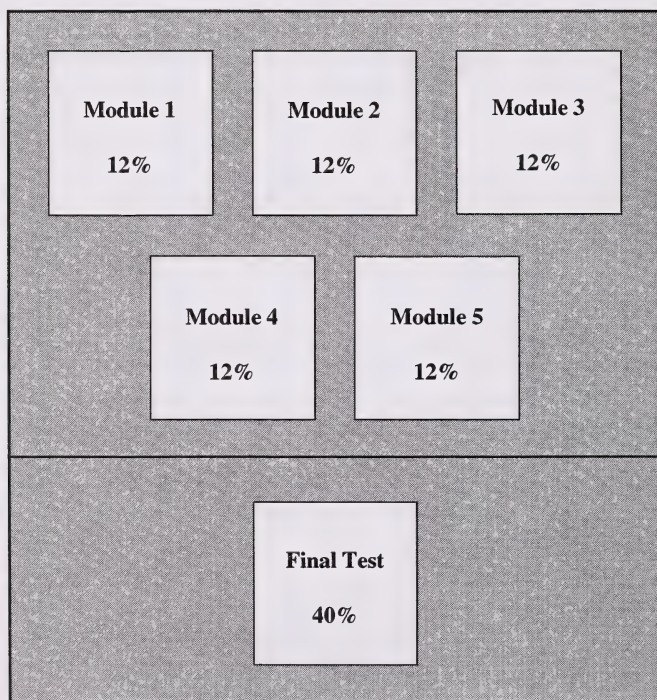
A chart or checklist could be used for recording responses.

Assignments

Each module contains separate booklets called the Assignment Booklets. These booklets are tools for assessing the knowledge or skills that the student has gained from the module. **The student's mark for the module may be based solely on the outcome of learning evident in the Assignment Booklets; however, you may decide to establish a value for other variables such as attitude or effort.** It is important that you establish at the beginning of the course and each module those outcomes that will be assessed, and ensure that all students clearly understand what is expected.

Formal Assessment

This Teacher's Guide includes a formal Final Test that can be photocopied for each member of the class. The test, closely linked to the learning outcomes stated in the Student Module Booklets, gives the teacher precise information concerning what each student can or cannot do. Answers, explanations, and marking guides are also included. The value of the Final Test and each module is the decision of the classroom teacher. Following is a suggestion only.



Note: The percent values given here, assume a student is enrolled in all five modules of Mathematics Preparation 10. A student may enrol in single modules or any combination of modules. The marking scheme must be adjusted appropriately.

Introducing Students to the System

Your initiation to these learning materials began with a basic survey of what was included and how the components varied. This same process should be repeated with the class. After the materials have been explored, a discussion might include the advantages and the disadvantages of learning independently or in small groups. The roles of the student and teacher should be analysed. The necessary progress checks and rules need to be addressed. Your introduction should motivate students and build a responsible attitude towards learning autonomously.

Skill Level

It is important for students to understand that there are certain skills that they will need in order to deal successfully with the course materials. Those skills are as follows:

- understanding and using instructional materials (table of contents, index, list of illustrations, appendices, bibliography, and glossary)
- interpreting maps, graphs, and charts
- using reference materials
- recognizing special symbols
- using a scientific calculator
- understanding and using different forms of media

Other general skills follow: using reliable study methods, outlining, and learning to read at a flexible rate.

To decide the level and amount of instruction needed to accommodate the varied levels among students, you may wish to prepare and administer skill inventories or pretests. If most students need help with a particular skill, you may want to plan a total-class instructional session. If only certain students lack a skill, you may want to set up a temporary skill group to help students who need it, or you may want to develop a skills file for this purpose.

Reading Level

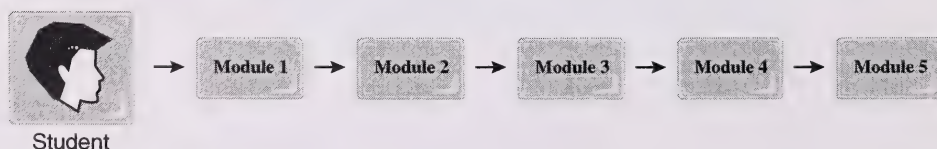
These course materials are largely print based, but poorer readers need not be discouraged. It is important that you assure students that these materials have been designed for easy reading. The authors have employed special strategies to reduce and control the reading level. Some of them are as follows:

- the conscious selection of vocabulary and careful structuring of sentences to keep the materials at an independent reading level
- the integration of activities, examples, and illustrations to break text into appropriate-sized chunks

- the inclusion of many kinds of organizers (advance, graphic, intermediate, concept mapping, post organizers) to help give students a structure for incorporating new concepts
- the recognition that vocabulary and concepts are basic to understanding the content and, thus, must be handled systematically (defined in context, signalled in marginal notes or footnotes, and cited in a glossary)
- the acknowledgement that background knowledge and experience play a vital role in comprehension
- the systematic inclusion of illustrations, videos, audiocassettes, and software to help poorer readers and visual learners
- a use of a variety of formats (paragraphs, lists, charts, etc.) to help struggling readers who do not absorb or retain main ideas easily in paragraph format
- the inclusion of media and activity choices to encourage an active rather than passive approach
- the development of instruction in a meaningful setting rather than in a contrived, workbook style
- the use of purposeful reading, viewing, and doing to produce better interpretation of the course materials
- the recognition that students need structured learning experiences when reading, viewing, or listening to instructional materials; and the acknowledgement that the recognized reading process provides such a structured framework by emphasizing the following phases: developing pupil readiness, determining the purpose, providing guided instruction and feedback, ensuring opportunity to reread or review if necessary, and including enrichment or extensions

To help make the learning package more readable, you can begin your course or module preparation by reading (viewing, listening to) all the related materials that are going to be used. You will need this solid understanding in order to link the content to the students' experiential base. This linking may be done through class brainstorming sessions concerning the topic or by using visuals and guided questions to predict what the course or modules might be about. Such a strategy helps poorer readers strengthen their abilities to be able to predict new vocabulary.

It is recommended that you have students begin with Module 1, because this module includes basic introductory information, and it is also recommended that you end with Module 5, because this module acts as a summary or culmination. A student may, however, complete individual modules of Mathematics Preparation 10 for 1 credit each.



Module 1: Exploration of Numbers

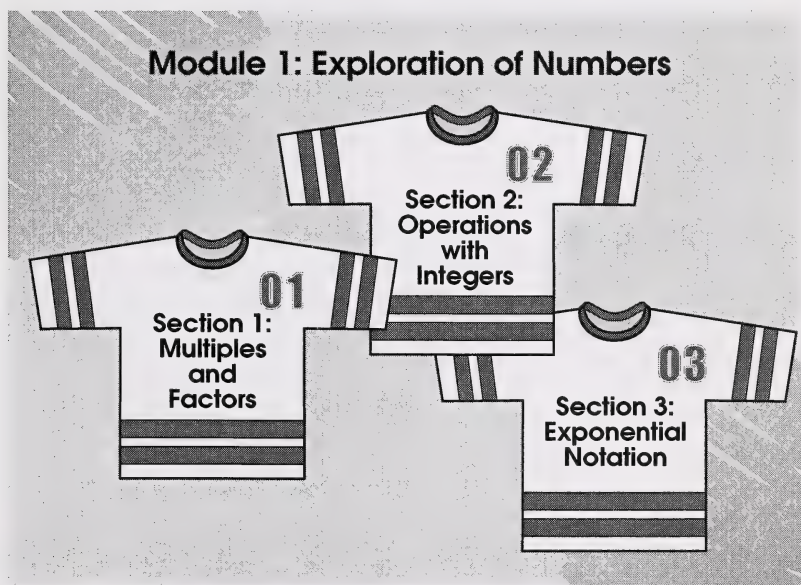
Overview

In this module, students review key number concepts previously covered in junior high mathematics, working with multiples, factors, integers, and exponents.

In Section 1, students first explore multiples, common multiples and lowest common multiples. Then they work with factors, common factors, and greatest common factors. In these topics, the students explore their practical uses, especially in working with fractions and divisibility.

In Section 2, students explore integers and their real world uses. They then work with adding, subtracting, multiplying and dividing integers, using positive and negative counters as models. They learn the sign rules applying to those operations.

In Section 3, students explore powers. They multiply and divide powers and study the power rules. The order of operations is reviewed and students work with a calculator. Scientific notation is then covered.



Assessment

The assessment of this module will be based on three assignments:

Assignment Booklet 1A	
Section 1 Assignment	30 marks
Assignment Booklet 1B	
Section 2 Assignment	39 marks
Assignment Booklet 1C	
Section 3 Assignment	<u>31 marks</u>
TOTAL	100 marks

Section 1: Multiples and Factors

Key Concepts

- multiples
- lowest common multiples
- common factors
- common multiples
- factors
- greatest common factors

The basic goals of this section are to ensure that students are able to find common multiples, common factors, lowest common multiples, and greatest common factors.

Section 1: Assignment Answer Key (30 marks)

1. 12, 24, 36, 48, 60, 72 (1 mark)

2. The denominators are 2, 3, and 15.

2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, (30), 32, ...

3, 6, 9, 12, 15, 18, 21, 24, 27, (30), 33, ...

15, (30), 45, 60, ...

The least common denominator is 30. (2 marks)

3. The multiples of 10 are 10, 20, 30, 40, ... Multiples can be found through repeated addition or by multiplying by a whole number. Because 2.5 is not a whole number, $2.5 \times 10 = 25$ is not considered a multiple of 10. (2 marks)

4. The family bought a winter coat, boots, and running shoes in January 2000.

9, 18, 27, 36, 45, 54, 63, (72), 81, ...

12, 24, 36, 48, 60, (72), 84, ...

24, 48, (72), 96, ...

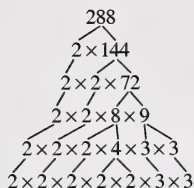
The family will have to buy all three items at the same time in 72 months (or 6 years). This will be in January 2006. (3 marks)

5.

124	
1	124
2	62
4	31

The factors are 1, 2, 4, 31, 62, and 124. (1 mark)

6.



$$28 = 2 \times 2 \times 7$$

$$288 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3$$

$$\begin{aligned}
 \text{GCF} &= 2 \times 2 \\
 &= 4
 \end{aligned}$$

$$\therefore \frac{28}{288} = \frac{7}{72} \quad (3 \text{ marks})$$

7. The GCF between 18 and 30 is needed.

$$18 = 2 \times 3 \times 3$$

$$30 = 2 \times 3 \times 5$$

$$\begin{aligned}
 \text{GCF} &= 2 \times 3 \\
 &= 6
 \end{aligned}$$

The most each t-shirt could have cost is \$6. (2 marks)

8.

	48
①	48
②	24
3	⑩ 16
④	12
~~5~~	
6	⑧
~~7~~	
~~8~~	~~6~~

	80
①	80
②	40
~~3~~	
④	20
5	⑩ 16
~~6~~	
~~7~~	
⑧	10
~~9~~	
~~10~~	~~8~~

The common factors of 48 and 80 are 1, 2, 4, 8, and 16. The greatest common factor is 16. (2 marks)

9. Rachel must have 32, 33, 34, 35, 36, 37, 38, 39, or 40 miniatures.

The possibilities 32, 34, 36, 38, and 40 may be eliminated because each is divisible by 2. (The miniatures cannot be arranged in rows of 2.)

~~32~~, 33, ~~34~~, 35, ~~36~~, 37, ~~38~~, 39, or ~~40~~

The possibilities 33 and 39 may be eliminated because each is divisible by 3. (The miniatures cannot be arranged in rows of 3.)

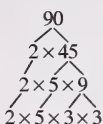
~~32~~, ~~33~~, ~~34~~, 35, ~~36~~, 37, ~~38~~, ~~39~~, or ~~40~~

The possibility 35 may be eliminated because it is divisible by 5. (The miniatures cannot be arranged in rows of 5.)

~~32~~, ~~33~~, ~~34~~, ~~35~~, ~~36~~, 37, ~~38~~, ~~39~~, or ~~40~~

Therefore, by elimination, Rachel must have 37 miniatures. **(4 marks)**

10. Examples will vary. The two numbers chosen must be relatively prime. The product of the two numbers must be the LCM of the numbers. Some examples are 5 and 3, 7 and 2, and 14 and 5. **(2 marks)**

- 11.
- 
- $2 \times 5 \times 3 \times 3$ **(1 mark)**

12. To be divisible by 4, the last two digits must be divisible by 4; to be divisible by 5, the last digit must be 0 or 5. Both conditions must be met.

Therefore, the number could be any 3-digit number ending with 00, 20, 40, 60, and 80. **(3 marks)**

13. 2, 4, 6, 8, 10, 12, 14, 16, ...

3, 6, 9, 12, 15, 18, 21, ...

4, 8, 12, 16, 20, ...

The least common multiple is 12. Therefore, the next time the brothers will go home in the same year will be 2011. **(4 marks)**

Section 2: Operations on Integers

Key Concepts

- defining integers
- adding and subtracting integers
- sign rules
- multiplying and dividing integers

The basic goals of this section are to ensure that students are able to

- compare and order integers
- use a scientific calculator to solve problems involving integers
- document and explain the calculator keying sequences used to perform operations on integers
- perform arithmetic operations with integers concretely, pictorially, and symbolically

Section 2: Assignment Answer Key (39 marks)

1. a. -15 (1 mark) b. -125 (1 mark) c. 12 or $+12$ (1 mark)
 d. 3 or $+3$ (1 mark) e. $-1\,000\,000$ (1 mark)

2. $-23^\circ, -17^\circ, -1^\circ, 3^\circ, 5^\circ$ (1 mark)

3.

Action	Opposite Action	
	Statement	Integer
ascending 20 m	descending 20 m	-20
credit of \$100	debit of \$100	-100
loss of 10 yards	gain of 10 yards	10 or $+10$
down 8 flights of stairs	up 8 flights of stairs	8 or $+8$
profit of \$25	loss of \$25	-25

(10 marks)

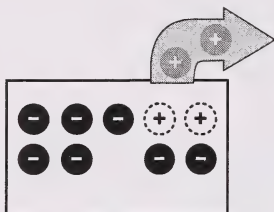
4. a. 3 or $+3$ (1 mark)
 b. 0 (1 mark)
 c. -5 (1 mark)
5. Begin with five negative counters.



You need to take away two positive counters, but there are no positive counters. So, you must add two zero pairs ($+$ $-$ $+$ $-$) so you have two positive counters to remove.



Now, it is possible to remove two positive counters, leaving seven negative counters.



$$\therefore (-5) - (+2) = -7 \quad (3 \text{ marks})$$

6.

Tim's Predicted Number of Tricks	Tricks Actually Obtained	Calculation	Score
1	3	$2 \times (-10) = -20$	-20
2	4	$2 \times (-10) = -20$	-20
3	0	$3 \times (-10) = -30$	-30
4	4	$20 + 4 \times 10 = 60$	60
5	1	$4 \times (-10) = -40$	-40
Total			-50

(4 marks)

7. a. $-20 + 50 = 30$

Paul now has \$30 in his bank account. (2 marks)

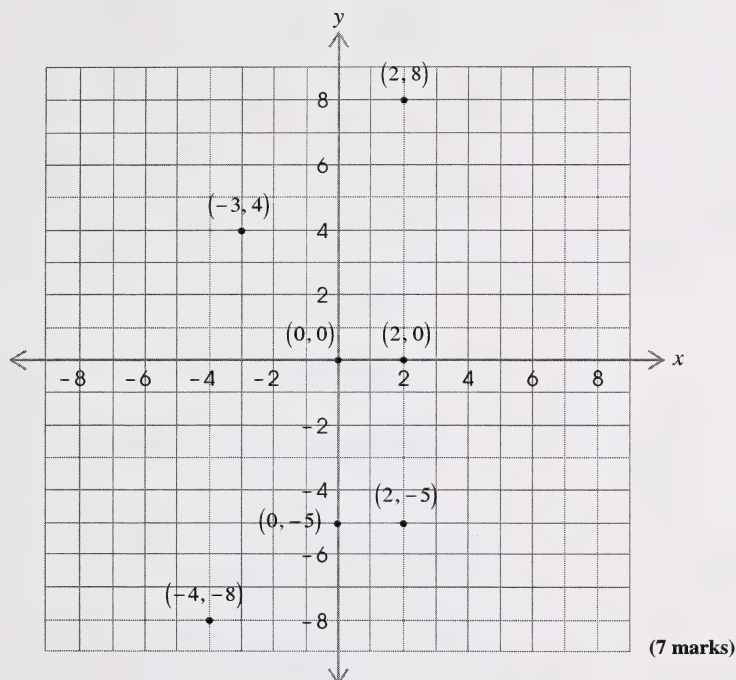
b. $18 + 7 = 25$

The temperature is 25°C . (2 marks)

c. $-50 + 10 = -40$

Sandy still owes her parents \$40. (2 marks)

8.



Section 3: Exponential Notation

Key Concepts

- definitions related to powers
- multiplying and dividing powers
- order of operations
- adding and subtracting powers
- power rules
- scientific notation

The basic goals of this section are to ensure that students are able to

- differentiate between principal square root (positive root) and negative square root of a number
- recognize and illustrate the meaning of a power, base, coefficient, and exponent, including rational numbers or variables as bases or coefficients
- explain and apply the exponent laws for powers with integral exponents
- document and explain the calculator keying sequences used to perform operations on square roots, exponents, and scientific notation
- illustrate and explain the order of operations
- use exponent laws to evaluate expressions with numerical bases
- understand and use the exponent laws to simplify expressions with variables as bases and use substitution to calculate a numerical value

Section 3: Assignment Answer Key (31 marks)

1. **Note to teacher:** Students may use a calculator to evaluate each expression.

$$\begin{aligned} \text{a. } 15^3 &= 15 \times 15 \times 15 \\ &= 3375 \quad (1 \text{ mark}) \end{aligned}$$

$$\begin{aligned} \text{b. } -4^6 &= -(4 \times 4 \times 4 \times 4 \times 4 \times 4) \\ &= -4096 \quad (1 \text{ mark}) \end{aligned}$$

$$\begin{aligned} \text{c. } (-5)^7 &= (-5) \times (-5) \times (-5) \times (-5) \times (-5) \times (-5) \times (-5) \\ &= -78\,125 \quad (1 \text{ mark}) \end{aligned}$$

$$\begin{aligned} \text{d. } 3^{-4} &= \left(\frac{1}{3}\right)^4 \\ &= \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \\ &= \frac{1}{81} \quad (1 \text{ mark}) \end{aligned}$$

$$\text{e. } 0.25^0 = 1 \quad (1 \text{ mark})$$

$$\begin{aligned} \text{f. } \left(\frac{1}{2}\right)^{-2} &= \left(\frac{2}{1}\right)^2 \\ &= \frac{2}{1} \times \frac{2}{1} \\ &= 4 \quad (1 \text{ mark}) \end{aligned}$$

$$\text{2. a. } 0.1 \times 0.1 \times 0.1 = 0.1^3 \quad (1 \text{ mark})$$

$$\text{b. } (-4) \times (-4) \times (-4) \times (-4) = (-4)^4 \quad (1 \text{ mark})$$

$$\text{c. } -(10 \times 10 \times 10 \times 10 \times 10) = -10^5 \quad (1 \text{ mark})$$

$$\begin{aligned} \text{3. a. } 5^6 \times 5^{-4} &= 5^{6+(-4)} \\ &= 5^2 \quad (1 \text{ mark}) \end{aligned}$$

$$\begin{aligned} \text{b. } (8^2)^3 &= 8^{2 \times 3} \\ &= 8^6 \quad (1 \text{ mark}) \end{aligned}$$

$$\begin{aligned} \text{c. } 0.8^4 \div 0.8^{-3} &= 0.8^{4-(-3)} \\ &= 0.8^{4+3} \\ &= 0.8^7 \quad (1 \text{ mark}) \end{aligned}$$

4. In order to write an expression as a single power, you need the same base. The numbers 12 and 4 cannot be written with common bases, so they cannot be written as a single power. (2 marks)

$$\begin{aligned} \text{5. a. } 4^2 - [6 - 2(7 - 1)] - 6^2 \div 4 &= 4^2 - [6 - 2(6)] - 6^2 \div 4 \\ &= 4^2 - (6 - 12) - 6^2 \div 4 \\ &= 4^2 - (-6) - 6^2 \div 4 \\ &= 16 + 6 - 36 \div 4 \\ &= 16 + 6 - 9 \\ &= 13 \quad (2 \text{ marks}) \end{aligned}$$

$$\begin{aligned} \text{b. } 5^2 + 6^2 \times 2 \div 3^2 + 14 &= 25 + 36 \times 2 \div 9 + 14 \\ &= 25 + 72 \div 9 + 14 \\ &= 25 + 8 + 14 \\ &= 47 \quad (2 \text{ marks}) \end{aligned}$$

$$\begin{aligned} \text{c. } \frac{3(8^2 - 6^2)}{3 - 9} &= \frac{3(64 - 36)}{3 - 9} \\ &= \frac{3(28)}{-6} \\ &= \frac{84}{-6} \\ &= -14 \quad (2 \text{ marks}) \end{aligned}$$

$$\begin{aligned} \text{d. } 4(7 - 2)^2 - (10 \div 2)^2 &= 4(5)^2 - 5^2 \\ &= 4(25) - 25 \\ &= 100 - 25 \\ &= 75 \quad (2 \text{ marks}) \end{aligned}$$

6. a. $9\,920\,000\text{ km}^2 = 9.92 \times 10^6\text{ km}^2$ (1 mark)
 b. $0.000\,000\,022\text{ cm} = 2.2 \times 10^{-8}\text{ cm}$ (1 mark)
 c. $0.000\,11\text{ cm} = 1.1 \times 10^{-4}\text{ cm}$ (1 mark)

$$\begin{aligned}
 7. \quad \frac{800}{5 \times 10^{-8}} &= \frac{8 \times 10^2}{5 \times 10^{-8}} \\
 &= (8 \div 5) \times (10^2 \div 10^{-8}) \\
 &= 1.6 \times 10^{2-(-8)} \\
 &= 1.6 \times 10^{10}
 \end{aligned}$$

The oil will cover $1.6 \times 10^{10}\text{ cm}^2$ on water. (3 marks)

$$\begin{aligned}
 8. \quad (6.0 \times 10^9) \times (6.3 \times 10^9) &= (6.0 \times 6.3) \times (10^9 \times 10^9) \\
 &= 37.8 \times 10^{18} \\
 &= 3.78 \times 10^{19}
 \end{aligned}$$

There are 3.78×10^{19} cells in the human body. (3 marks)

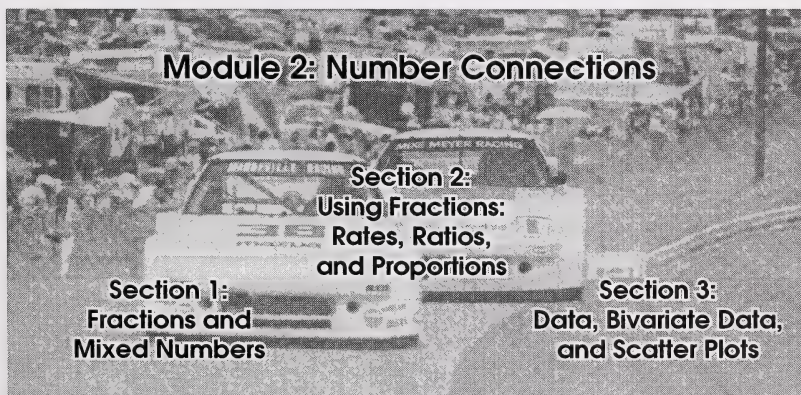
Module 2: Number Connections

Overview

Section 1 of this module reintroduces students to fractions, their several forms, decimal representation, and associated terminology. Students gain proficiency in converting between fractions and decimal numerals; generating equivalent fractions; handling common and mixed fractions; and applying the addition, subtraction, multiplication, and division algorithms. Every attempt is made to create meaning through motivating examples, diagrams and models, and real-world problem situations.

Section 2 links fractions to rates, ratios, proportions, and percents. Again, the emphasis is on developing an understanding within problem solving and familiar contexts.

Section 3 introduces students to the collection and analysis of numerical data. Students propose and conduct experiments, graph bivariate data to produce scatter plots, and use lines of best fit to recognize trends and make predictions based on these scatter plots.



Assessment

The assessment of this module will be based on three assignments:

Assignment Booklet 2A	
Section 1 Assignment	39 marks
Assignment Booklet 2B	
Section 2 Assignment	30 marks
Assignment Booklet 2C	
Section 3 Assignment	31 marks
TOTAL	100 marks

Section 1: Fractions and Mixed Numbers

Key Concepts

- common and mixed fractions
- proper and improper fractions
- decimal representation of fractions
- terminating and repeating decimals
- equivalent fractions
- addition, subtraction, multiplication, and division of fractions

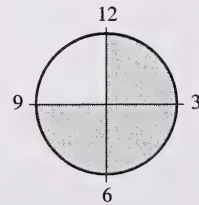
The basic goals of this section are to ensure that students are able to

- demonstrate and explain the meaning of proper and improper fractions
- convert among fractions and decimals
- add, subtract, multiply, and divide fractions concretely, pictorially, and symbolically
- estimate answers arising from the combination of fractions
- use a variety of methods to solve problems involving fractions

Section 1: Assignment Answer Key (39 marks)

1. You could also describe $\frac{3}{4}$ h as 45 min.

Imagine a clock divided into quarters. Each quarter represents 15 min. The shaded area represents $\frac{3}{4}$ h or 45 min. (2 marks)



2. Equivalent fractions may vary. Here are some equivalent fractions:

$$\frac{5}{12} = \frac{10}{24} = \frac{15}{36} = \frac{20}{48} = \frac{25}{60}$$

Equivalent fractions are used to find common denominators when adding and subtracting. (4 marks)

3.

Proper Fraction	Improper Fraction	Mixed Number	Decimal
	$\frac{16}{3}$	$5\frac{1}{3}$	$5.\bar{3}$
	$\frac{11}{4}$	$2\frac{3}{4}$	2.75
$\frac{2}{5}$			0.4
	$\frac{5}{4}$	$1\frac{1}{4}$	1.25
$\frac{7}{9}$			$0.\bar{7}$

(8 marks)

4. a. To determine the number of hours Jennifer worked, add the areas of A, B, C, and D. **Note:** Students may label the areas differently.

$$\begin{aligned}\text{Area } A &= 5 \times 7 \\ &= 35\end{aligned}$$

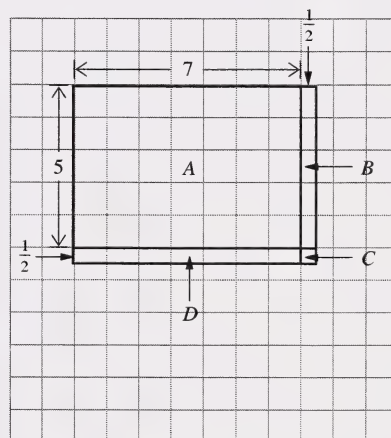
$$\begin{aligned}\text{Area } B &= 5 \times \frac{1}{2} \\ &= \frac{5}{2}\end{aligned}$$

$$\begin{aligned}\text{Area } C &= \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{4}\end{aligned}$$

$$\begin{aligned}\text{Area } D &= \frac{1}{2} \times 7 \\ &= \frac{7}{2}\end{aligned}$$

$$\begin{aligned}\text{Total area} &= 35 + \frac{7}{2} + \frac{5}{2} + \frac{1}{4} \\ &= 35 + \frac{12}{2} + \frac{1}{4} \\ &= 35 + 6 + \frac{1}{4} \\ &= 41\frac{1}{4}\end{aligned}$$

Jennifer worked $41\frac{1}{4}$ h. (6 marks)



$$\begin{aligned}
 \text{b. } 7\frac{1}{2} \times 5\frac{1}{2} &= \frac{15}{2} \times \frac{11}{2} \\
 &= \frac{15 \times 11}{2 \times 2} \\
 &= \frac{165}{4} \\
 &= 41\frac{1}{4}
 \end{aligned}$$

Jennifer worked $41\frac{1}{4}$ h. (2 marks)

$$\begin{aligned}
 5. \text{ a. } 1\frac{1}{2} + \frac{3}{4} &= \frac{3}{2} + \frac{3}{4} \\
 &= \frac{6}{4} + \frac{3}{4} \\
 &= \frac{9}{4} \text{ or } 2\frac{1}{4} \quad (1 \text{ mark})
 \end{aligned}$$

$$\begin{aligned}
 \text{b. } \frac{7}{4} - \frac{1}{3} &= \frac{21}{12} - \frac{4}{12} \\
 &= \frac{17}{12} \text{ or } 1\frac{5}{12} \quad (1 \text{ mark})
 \end{aligned}$$

$$\begin{aligned}
 \text{c. } \frac{5}{6} \div \frac{2}{3} &= \frac{5}{6} \times \frac{3}{2} \\
 &= \frac{5 \times \cancel{3}}{\cancel{6} \times 2} \\
 &= \frac{5}{4} \text{ or } 1\frac{1}{4} \quad (1 \text{ mark})
 \end{aligned}$$

$$\begin{aligned}
 \text{d. } 4\frac{1}{3} - 1\frac{5}{6} &= \frac{13}{3} - \frac{11}{6} \\
 &= \frac{26}{6} - \frac{11}{6} \\
 &= \frac{15}{6} \\
 &= \frac{5}{2} \text{ or } 2\frac{1}{2} \quad (1 \text{ mark})
 \end{aligned}$$

$$\begin{aligned}
 \text{e. } 2\frac{1}{3} \times 4\frac{1}{2} &= \frac{7}{3} \times \frac{9}{2} \\
 &= \frac{7 \times \cancel{9}^3}{\cancel{3} \times 2} \\
 &= \frac{21}{2} \text{ or } 10\frac{1}{2} \quad (1 \text{ mark})
 \end{aligned}$$

6. Step 2 corresponds to finding a common denominator. (1 mark)

$$\begin{aligned}
 7. \quad \frac{1}{2} \times 5 &= \frac{1 \times 5}{2} \\
 &= \frac{5}{2} \text{ or } 2\frac{1}{2}
 \end{aligned}$$

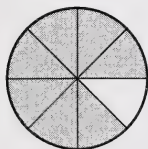
The total amount of homework expected is $\frac{5}{2}$ h or $2\frac{1}{2}$ h. (1 mark)

$$\begin{aligned}
 8. \quad \frac{1}{3} + 1\frac{1}{2} &= \frac{1}{3} + \frac{3}{2} \\
 &= \frac{2}{6} + \frac{9}{6} \\
 &= \frac{11}{6} \text{ or } 1\frac{5}{6}
 \end{aligned}$$

The recipe requires $1\frac{5}{6}$ cups of sugar. (1 mark)

$$\begin{aligned}
 9. \quad \frac{3}{4} \text{ of } \frac{7}{8} &= \frac{3}{4} \times \frac{7}{8} \\
 &= \frac{3 \times 7}{4 \times 8} \\
 &= \frac{21}{32}
 \end{aligned}$$

The shaded area represents $\frac{7}{8}$ of George's CD collection.



The answer is less than $\frac{3}{4}$ because George is working with less than the whole collection (only $\frac{7}{8}$ of the collection). (2 marks)

10. Michael picked more since $\frac{7}{8} > \frac{3}{4}$.

$$\begin{aligned}
 \frac{7}{8} - \frac{3}{4} &= \frac{7}{8} - \frac{6}{8} \\
 &= \frac{1}{8}
 \end{aligned}$$

He picked $\frac{1}{8}$ of an ice cream bucket more than Christine picked. (2 marks)

$$11. \quad 4 \overline{)14}^3 \\
 \underline{12} \\
 2$$

They will need 4 cars, since 3 cars are not enough. (1 mark)

12. a. Gerda needs 5 boxes. (1 mark)

b. number of bulbs in fifth box $= 96 - 36$
 $= 60$

$$\frac{60}{96} = \frac{5}{8}$$

The box is $\frac{5}{8}$ filled. (1 mark)

13. a. $1 - \left(\frac{2}{3} + \frac{1}{6}\right) = 1 - \left(\frac{4}{6} + \frac{1}{6}\right)$
 $= 1 - \frac{5}{6}$
 $= \frac{6}{6} - \frac{5}{6}$
 $= \frac{1}{6}$

Gloria budgets $\frac{1}{6}$ of her money for entertainment. (1 mark)

b. $120 \times \frac{1}{6} = \frac{120}{6}$
 $= 20$

She budgets \$20 for bus fare. (1 mark)

Section 2: Using Fractions: Rates, Ratios, and Proportions

Key Concepts

- rate
- ratio
- percent
- proportion

The basic goals of this section are to ensure that students are able to

- understand the meaning of rate, ratio, percent, and proportion and apply these concepts to solve problems
- express rates and ratios in equivalent forms
- convert among fractions, decimals, and percents to solve problems
- use a variety of methods to solve problems involving ratios, rates, and percents

Section 2: Assignment Answer Key (30 marks)

1. a. The ratio 1 to 4 represents the number of cats compared to the number of dogs. (1 mark)

- b. The ratio $\frac{5}{7}$ represents the total number of cats and dogs compared to the number of condominiums. (1 mark)

c. $\frac{4}{7} = \frac{36}{63}$

There would be 36 dogs in the complex. (2 marks)

2. a. $\frac{58\%}{56\%} = \frac{58}{56}$
 $= \frac{29}{28}$ (2 marks)

- b. $100\% - 58\% = 42\%$ (2 marks)

The percentage of women that exercise less than three times weekly is 42%.

c. $42\% = 0.42$ $42\% = \frac{42}{100}$
 $= \frac{21}{50}$

(2 marks)

d. $\frac{56}{100} = \frac{224}{400}$

There would be 224 men that exercise three or more times weekly. (2 marks)

3. $\frac{500}{4} = \frac{125}{1}$ (2 marks)

The rate is 125 mg/cup.

4. Answers will vary. A sample answer is given.

$$\frac{\text{U.S. \$1.00}}{\text{CDN \$1.38}} = \frac{\text{U.S. \$100}}{\text{CDN \$147}} = \frac{\text{U.S. \$1000}}{\text{CDN \$1470}} \quad (2 \text{ marks})$$

5. a. Find the fraction of students that chose either the museum or the art gallery.

$$1 - \frac{3}{5} = \frac{5}{5} - \frac{3}{5} \\ = \frac{2}{5}$$

Now, find the fraction of students that chose the museum.

$$\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$$

Finally, calculate the percent of students that chose the museum.

$$\frac{3}{5} = \frac{20}{100} \\ = 20\%$$

Therefore, 20% of the students chose the museum. **(2 marks)**

b. $\frac{3}{5} = \frac{720}{1200}$

Therefore, 720 students chose the waterslide. **(2 marks)**

6. $\frac{21}{25} = \frac{84}{100}$

Derek answered 84% of the questions correctly. **(1 mark)**

7. a. $\frac{300}{1250} = \frac{24}{100}$

Soon-Li received 24% of the total sales. **(2 marks)**

b. Number of hours worked $= 1\frac{1}{2} + 4\frac{1}{2}$

$$= \frac{3}{2} + \frac{9}{2} \\ = \frac{12}{2} \\ = 6$$

$$\therefore \text{hourly rate} = \frac{300}{6} \\ = 50$$

On that particular evening, Soon-Li earned \$50/h. **(2 marks)**

8. a. $\frac{6}{26}$ or $\frac{3}{13}$ (1 mark)

b. $\frac{6}{32} = \frac{18.75}{100}$

Therefore, 18.75% of the children wear glasses. (2 marks)

9. $\frac{120}{100} = \frac{300}{250}$

The airline has reserved seats for 300 people, but have only 250 seats available. Therefore, they need to make arrangements for $300 - 250 = 50$ people. (2 marks)

Section 3: Data, Bivariate Data, and Scatter Plots

Key Concepts

- statistics
- experiment
- data
- bivariate data
- scatter plot
- line of best fit
- interpolation and extrapolation

The basic goals of this section are to ensure that students are able to

- read and interpret graphs that are provided
- create scatter plots for discrete and continuous data
- interpret a scatter plot to determine if there is an apparent relationship
- determine the line of best fit from a scatter plot for an apparent linear relationship by inspection
- draw and justify conclusions from the line of best fit by interpolation and extrapolation

Section 3: Assignment Answer Key (31 marks)

1. a. The data in the pictograph is arranged alphabetically by team name. (1 mark)
- b. Let x represent the percentage. Set up an equivalent ratio, and solve.

$$\frac{5}{31} = \frac{x}{100}$$

$$x \doteq 16.1$$

The Edmonton Oilers won approximately 16% of the Stanley Cups from 1960 to 1990. (2 marks)

- c. Answers may vary. Sample answers are given.

$$\frac{12}{4} = \frac{3}{1} \quad 12:4 = 3:1 \quad 12 \text{ to } 4 = 3 \text{ to } 1 \quad (2 \text{ marks})$$

- d. Answers may vary. Sample answers are given.

$$\frac{9}{22} \quad 9:22 \quad 9 \text{ to } 22 \quad (2 \text{ marks})$$

2. Answers will vary. Sample answers are given.

- a. The distance travelled is the independent variable, and the cost of the trip is the dependent variable.

You would expect a relationship between these variables because the further you go in a taxicab, the more it costs. You would expect the points plotted to rise to the right.

(3 marks)

- b. The student may state that there is no correlation. You would not expect any relationship because some people have enough spare time to watch a lot of television and read a lot. Other people may prefer to watch television and not read much or read a lot and not watch television much. More details are needed before a relationship can be discerned.

The student may state that it shows a negative correlation. It is feasible that the number of books read decreases with the amount of television watched. If you spend hours watching television, you may not have time to read; if you spend hours reading, you will not have time to watch television. Therefore, the points plotted will fall to the right. It is not clear which is the independent variable and which is the dependent variable.

(3 marks)

- c. The student may state that there is no correlation. There are so many deals offered by telephone companies now that there is no correlation. Some people talk long distance on the phone frequently and pay a fixed amount, while other people pay a fixed amount and do not talk much. The points will show no apparent relationship.

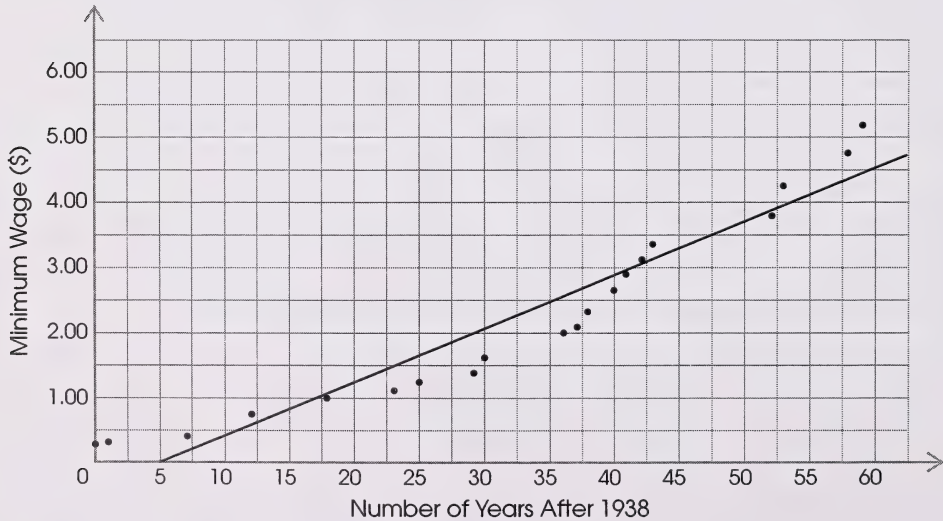
The student may state that there is a positive correlation. If you do not use any of the deals offered by telephone companies, the points plotted would probably rise to the right. The more you talk, the more you pay. If you do not phone long distance, the cost will be nothing. In this case, the number of minutes spent on the phone is the independent variable, and the cost is the dependent variable.

(3 marks)

3. a. The independent variable is the number of years since 1938. (1 mark)

b.

Federal Hourly Minimum Wage



(4 marks)

- c. Answers for the minimum wage in 1986 will vary depending on the student's line of best fit.

From the graph in question 3.b., the minimum wage in 1986 was approximately \$3.50. (2 marks)

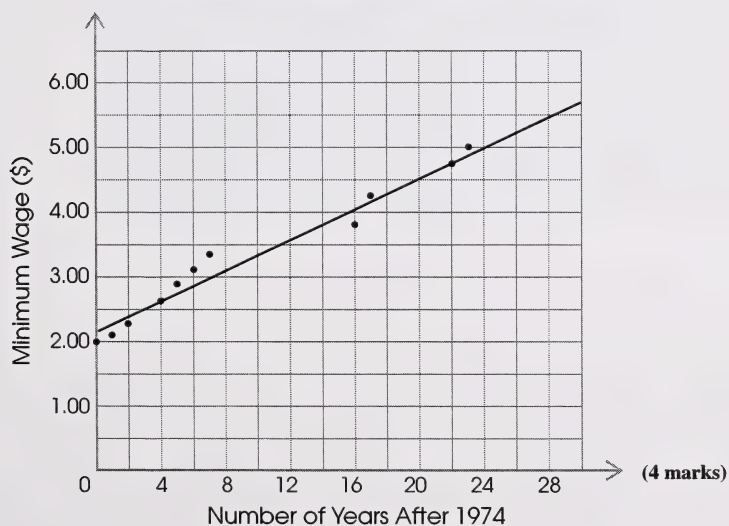
d.

Year	Number of Years	Minimum Wage
1974	0	2.00
1975	1	2.10
1976	2	2.30
1978	4	2.65
1979	5	2.90
1980	6	3.10
1981	7	3.35
1990	16	3.80
1991	17	4.25
1996	22	4.75
1997	23	5.15

(2 marks)

e.

Federal Hourly Minimum Wage



- f. The new line would be a better aid for your argument. Compare the minimum wage for 2000 on the first graph and second graph.

On the first graph, the year 2000 is year 62. Extrapolation on the first graph will give a minimum wage of about \$4.70 in the year 2000.

On the second graph, the year 2000 is year 26. Extrapolation on the second graph will give a minimum wage of about \$5.25 in the year 2000. **(2 marks)**

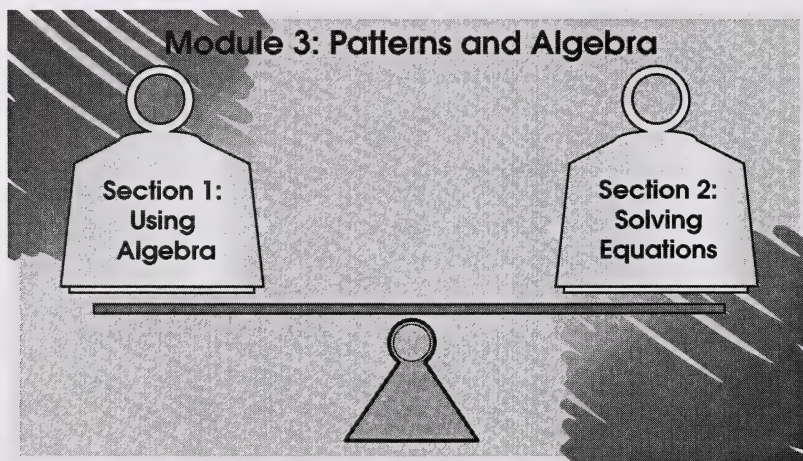
Module 3: Patterns and Algebra

Overview

In this module, students review the use of variables, algebraic expressions, and equations to model problem situations.

In Section 1, students translate between verbal and symbolic forms, evaluate expressions, graph simple functions, and apply a number of formulas—including those for surface area and volume of prisms, cylinders, pyramids, and cones. In their investigation of area and volume, students gain insight into the effects of scaling factors.

In Section 2, students revisit solving equations. Students apply balance models and pencil-and-paper techniques to solve and verify equations. Students apply these techniques to solve a variety of word problems.



Assessment

The assessment of this module will be based on three assignments:

Assignment Booklet 3A	
Section 1 Assignment	25 marks
Assignment Booklet 3B	
Section 2 Assignment	45 marks
Final Module Assignment	<u>30 marks</u>
TOTAL	100 marks

Section 1: Using Algebra

Key Concepts

- variables
- graphing
- algebraic expressions
- surface area
- algebraic equations
- volume
- formula

The basic goals of this section are to ensure that students are able to

- translate between oral and written expressions and equivalent algebraic expressions
- generalize a pattern using algebra and verify by substitution
- given a first-degree equation, substitute numbers for variables and graph and analyse the relation
- estimate, measure, and calculate the surface area and volume of any right prism, cylinder, pyramid, or cone
- demonstrate that many rectangles are possible for a given perimeter or a given area
- solve problems using different strategies

Section 1: Assignment Answer Key (25 marks)

1. a. $3n + 5$ (1 mark)

b. $2w$ (1 mark)

c. $\frac{I}{3}$ (1 mark)

2. a. Let C be the cost (in dollars), and let t be the time (in hours).

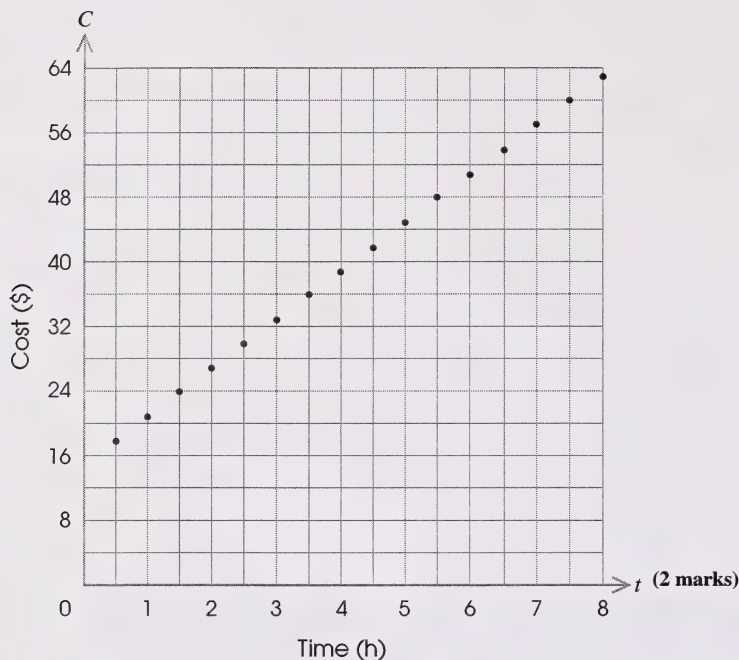
$$\therefore C = 15 + 6t \quad (1 \text{ mark})$$

b. Substitute $t = 4$ into the formula..

$$\begin{aligned} C &= 15 + 6t \\ &= 15 + 6(4) \\ &= 39 \end{aligned}$$

It would cost \$39 to rent a bicycle for 4 h. (1 mark)

c.



Note to teacher: The points are not joined because the bikes are rented in half-hour intervals. Accept a step function or joined points as well.

3. a. If the area is 64 square units and the length is 4 units, the width must be $\frac{64}{4} = 16$ units. Therefore, the perimeter is $2(4) + 2(16) = 40$ units.

If the area is 84 square units and the length is 6 units, the width must be $\frac{84}{6} = 14$ units. Therefore, the perimeter is $2(6) + 2(14) = 40$ units.

If the area is 96 square units and the length is 8 units, the width must be $\frac{96}{8} = 12$ units. Therefore, the perimeter is $2(8) + 2(12) = 40$ units.

All of the rectangles have a perimeter of 40 units. (2 marks)

- b. The ordered pair, $(10, 100)$, shows that the greatest area possible of all rectangles with a perimeter of 40 units is 100 square units. This ordered pair also shows that the greatest area occurs when the length of one side is 10 units. In this case, the rectangle is a square. (1 mark)

4. Calculate the areas of the top and bottom faces.

$$\begin{aligned} A &= \pi r^2 \\ &= \pi(3)^2 \\ &\doteq 28.27 \end{aligned}$$

The area of each face is approximately 28.27 cm^2 . Together, the top and bottom faces have an area of $2 \times 28.27 \doteq 56.54 \text{ cm}^2$.

Calculate the area of the curved surface.

$$\begin{aligned} A &= \ell w \\ &= (2\pi r)(90) \\ &= 2\pi(3)(90) \\ &\doteq 1696.46 \end{aligned}$$

Calculate the surface area of the tube.

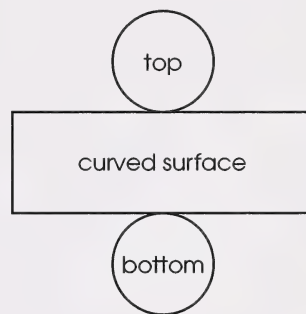
$$\begin{aligned} s &\doteq 56.54 + 1696.46 \\ &\doteq 1753.0 \end{aligned}$$

The surface area of the tubes is about 1753.0 cm^2 .

Calculate the volume.

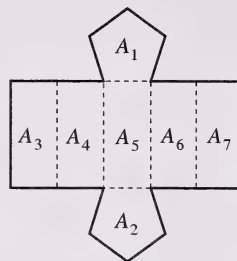
$$\begin{aligned} V &= BH \\ &= (\pi r^2)(90) \\ &= \pi(3)^2(90) \\ &\doteq 2544.7 \end{aligned}$$

The volume of each tube is about 2544.7 cm^3 . **(4 marks)**



5. Calculate the areas of the top and bottom faces (A_1 and A_2). **Note:** $A_1 = A_2$

$$\begin{aligned}
 A_1 &= \frac{nsa}{2} \\
 &= \frac{5 \left(\frac{35}{2} \right) (48.2)}{1} \\
 &= 8435
 \end{aligned}$$



The area of each face is 8435 cm^2 . Together, the top and bottom faces have an area of $2 \times 8435 = 16\,870 \text{ cm}^2$.

Calculate the area of each side. **Note:** $A_3 = A_4 = A_5 = A_6 = A_7$

$$\begin{aligned}
 A_3 &= \ell w \\
 &= 70(10) \\
 &= 700
 \end{aligned}$$

The area of each side is 700 cm^2 . Together, the area of all the sides is $5 \times 700 = 3500 \text{ cm}^2$.

Calculate the surface area of the brick.

$$\begin{aligned}
 S &= 16\,870 + 3500 \\
 &= 20\,370
 \end{aligned}$$

The surface area of the brick is $20\,370 \text{ cm}^2$.

Calculate the volume.

$$\begin{aligned}
 V &= BH \\
 &= \left(\frac{nsa}{2} \right) (10) \\
 &= (8435)(10) \\
 &= 84\,350
 \end{aligned}$$

The volume of the brick is $84\,350 \text{ cm}^3$. **(4 marks)**

6. The cylindrical cup can hold 3 times more water. **(1 mark)**

7. a. $\text{length} = 7 \times 4$
 $= 28 \text{ cm}$

$\text{width} = 7 \times 3$
 $= 21 \text{ cm}$

The box will have a length of 28 cm and a width of 21 cm. **(1 mark)**

b. $V = \ell wh$
 $= (28)(21)(12)$
 $= 7056$

The box will have a volume of 7056 cm^3 . **(1 mark)**

c. $\text{Volume of cans} = 12 \times \text{volume of 1 can}$
 $= 12 \times (\pi r^2) h$
 $= (12)(\pi)(3.5)^2(12)$
 $\doteq 5541.8$

$\text{Space not used} = \text{Volume of box} - \text{Volume of cans}$
 $= 7056 - 5541.8$
 $= 1514.2 \text{ cm}^3$

The space not used is about 1514 cm^3 . **(2 marks)**

d. New Box

$w = 14$, $\ell = 21$, and $h = 24$

$\therefore S = 2(14 \times 21) + 2(14 \times 24) + 2(21 \times 24)$
 $= 2268 \text{ cm}^2$

Old Box

$w = 21$, $\ell = 28$, and $h = 12$

$S = 2(21 \times 28) + 2(21 \times 12) + 2(28 \times 12)$
 $= 2352 \text{ cm}^2$

The new box requires less cardboard. **(2 marks)**

Section 2: Solving Equations

Key Concepts

- equation
- inverse
- solution
- verification
- opposite operation

The basic goals of this section are to ensure that students are able to

- illustrate the solution process for first degree, single-variable equations using concrete materials or diagrams
- use pencil-and-paper techniques to solve and verify first-degree, single-variable equations of the following forms:

$$x + a = b$$

$$ax + b = c$$

$$a(bx + c) = d(ex + f)$$

$$\frac{x}{a} = b$$

$$ax = b + cx$$

$$\frac{a}{x} = b$$

$$ax = b$$

$$a(x + b) = c$$

$$\frac{x}{a} + b = c$$

$$ax + b = cx + d$$

- model, solve, and verify problem situations using first-degree, single-variable equations

Section 2: Assignment Answer Key (45 marks)

1. a. $7x + 3 = -18$

$$7x + 3 - 3 = -18 - 3$$

$$7x = -21$$

$$\frac{7x}{7} = \frac{-21}{7}$$

$$x = -3$$

(2 marks)

Check

LS	RS
$7(-3) + 3$	-18
$= -21 + 3$	
$= -18$	
LS	= RS

b. $-3x - 8 = 6x + 10$

$$-3x - 8 + 8 = 6x + 10 + 8$$

$$-3x = 6x + 18$$

$$-3x - 6x = 6x + 18 - 6x$$

$$-9x = 18$$

$$\frac{-9x}{-9} = \frac{18}{-9}$$

$$x = -2$$

(2 marks)

Check

LS	RS
$-3(-2) - 8$	$6(-2) + 10$
$= 6 - 8$	$= -12 + 10$
$= -2$	$= -2$
LS	= RS

$$\begin{aligned}
 \text{c. } 4(x-1) &= 3(2x-4) \\
 4x-4 &= 6x-12 \\
 4x-4+4 &= 6x-12+4 \\
 4x &= 6x-8 \\
 4x-6x &= 6x-8-6x \\
 -2x &= -8 \\
 \frac{-2x}{-2} &= \frac{-8}{-2} \\
 x &= 4
 \end{aligned}$$

(2 marks)

$$\begin{aligned}
 \text{d. } \frac{5-2x}{6} &= \frac{1}{2} \\
 \frac{1}{8} \left(\frac{5-2x}{\frac{8}{1}} \right) &= \frac{3}{8} \left(\frac{1}{\frac{8}{1}} \right) \\
 5-2x &= 3 \\
 5-2x-5 &= 3-5 \\
 -2x &= -2 \\
 \frac{-2x}{-2} &= \frac{-2}{-2} \\
 x &= 1
 \end{aligned}$$

(2 marks)

$$\begin{aligned}
 \text{e. } 2(x-3) &= 14 \\
 2x-6 &= 14 \\
 2x-6+6 &= 14+6 \\
 2x &= 20 \\
 \frac{2x}{2} &= \frac{20}{2} \\
 x &= 10
 \end{aligned}$$

(2 marks)

$$\begin{aligned}
 \text{f. } 6.2x-4.8 &= -26.5 \\
 6.2x-4.8+4.8 &= -26.5+4.8 \\
 6.2x &= -21.7 \\
 \frac{6.2x}{6.2} &= \frac{-21.7}{6.2} \\
 x &= -3.5
 \end{aligned}$$

(2 marks)

Check

LS	RS
$4(x-1)$	$3(2x-4)$
$= 4(4-1)$	$= 3[2(4)-4]$
$= 4(3)$	$= 3(8-4)$
$= 12$	$= 3(4)$
	$= 12$
LS	= RS

Check

LS	RS
$\frac{5-2x}{6}$	$\frac{1}{2}$
$= \frac{5-2(1)}{6}$	
$= \frac{5-2}{6}$	
$= \frac{3}{6} = \frac{1}{2}$	
LS	= RS

Check

LS	RS
$2(x-3)$	14
$= 2(10-3)$	
$= 2(7)$	
$= 14$	
LS	= RS

Check

LS	RS
$6.2x-4.8$	-26.5
$= 6.2(-3.5)-4.8$	
$= -26.5$	
LS	= RS

2. a. If $AB = BC$, then

$$\begin{aligned}
 x &= 2x - 6 \\
 x - 2x &= 2x - 6 - 2x \\
 -1x &= -6 \\
 \frac{-1x}{-1} &= \frac{-6}{-1} \\
 x &= 6
 \end{aligned}$$

The triangle is isosceles if $x = 6$. (2 marks)

- b. The triangle could never be equilateral because side AB is x and side AC is $x + 2$. This means that side AC will always be 2 units more than side AB . Therefore, these sides can never be equal, a necessary condition for an equilateral triangle. (1 mark)

3. a.	LS	RS
	$2(x-1)$	12
	$= 2(5-1)$	
	$= 2(4)$	
	$= 8$	
	LS	RS

(1 mark)

- b. The student should add 2 to both sides instead of adding 2 to the left-hand side and subtracting 2 from the right-hand side. In Step 2, the student has $2x - 2$ on the left side. To get rid of -2 , you need to add 2 to both sides of the equation. (1 mark)

c. Method 1

$$\begin{aligned}
 2(x-1) &= 12 \\
 2x - 2 &= 12 \\
 2x - 2 + 2 &= 12 + 2 \\
 2x &= 14 \\
 \frac{2x}{2} &= \frac{14}{2} \\
 x &= 7
 \end{aligned}$$

(1 mark)

Method 2

$$\begin{aligned}
 2(x-1) &= 12 \\
 \frac{2(x-1)}{2} &= \frac{12}{2} \\
 x-1 &= 6 \\
 x-1+1 &= 6+1 \\
 x &= 7
 \end{aligned}$$

4. a. $A = \frac{1}{2}bh$

$$2(A) = 2\left(\frac{1}{2}bh\right)$$

$$\frac{2A}{h} = \frac{bh}{h}$$

$$\frac{2A}{h} = b$$

$$b = \frac{2A}{h} \quad (2 \text{ marks})$$

b. $V = \frac{1}{3}\pi r^2 h$

$$3(V) = 3\left(\frac{1}{3}\pi r^2 h\right)$$

$$\frac{3V}{\pi r^2} = \frac{\pi r^2 h}{\pi r^2}$$

$$\frac{3V}{\pi r^2} = h$$

$$h = \frac{3V}{\pi r^2} \quad (2 \text{ marks})$$

c.

$$A = \frac{(b_1 + b_2)h}{2}$$

$$2(A) = 2 \left[\frac{(b_1 + b_2)h}{2} \right]$$

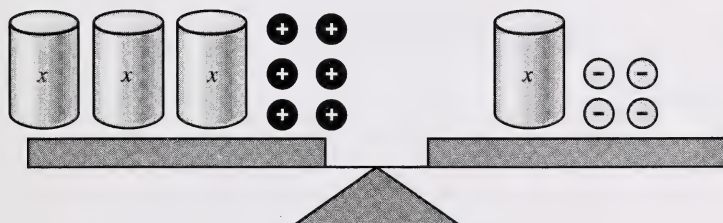
$$2A = (b_1 + b_2)h$$

$$\frac{2A}{(b_1 + b_2)} = \frac{(b_1 + b_2)h}{(b_1 + b_2)}$$

$$\frac{2A}{b_1 + b_2} = h$$

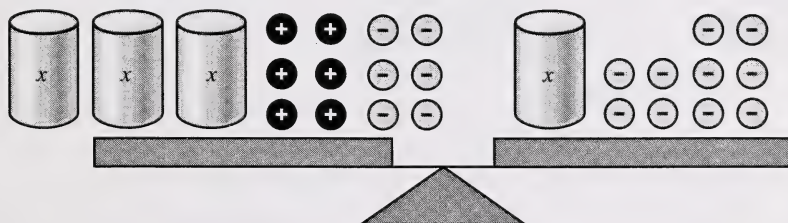
$$h = \frac{2A}{b_1 + b_2} \quad (2 \text{ marks})$$

5. Model the equation.



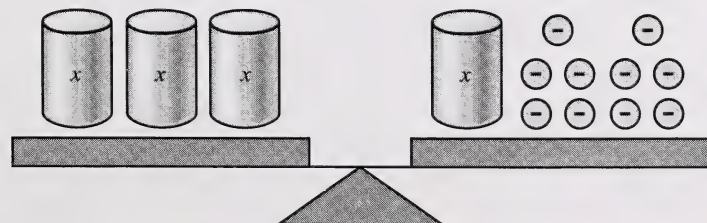
$$3x + 6 = x - 4$$

Isolate $3x$ on the left side by adding -6 to both sides.



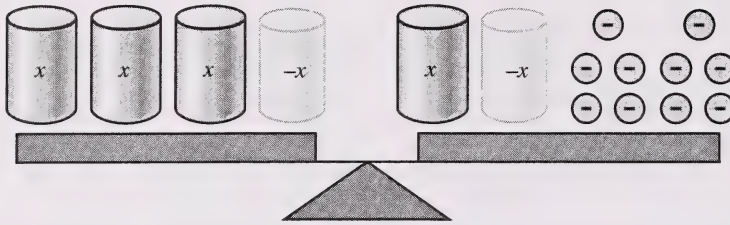
$$3x + 6 - 6 = x - 4 - 6$$

Simplify by removing any zero pairs. This is the result.



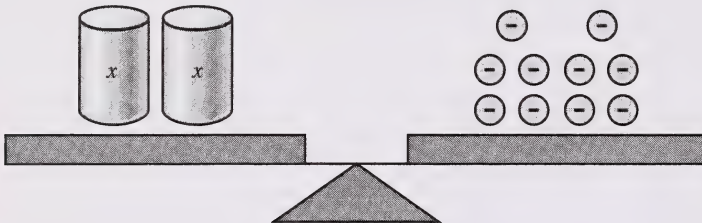
$$3x = x - 10$$

Isolate -10 on the right side by adding $-x$ to both sides.



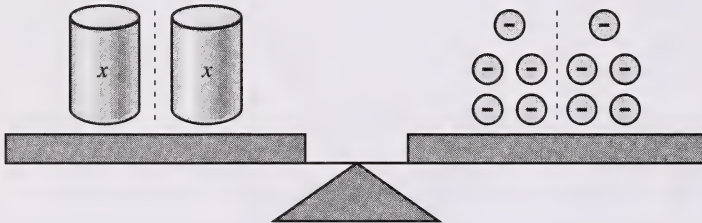
$$3x - x = x - 10 - x$$

Simplify by removing any zero pairs. This is the result.



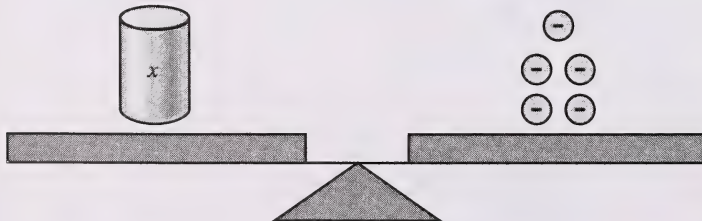
$$2x = -10$$

Isolate x by dividing each side into two groups.



$$\frac{2x}{2} = \frac{-10}{2}$$

Solve the equation by keeping one of the groups from each side of the equation.



$$x = -5$$

$\therefore x = -5$ (5 marks)

6. a. Let x be the amount spent on non-food items. Therefore, $4x$ will be the amount spent on food.

$$\therefore x + 4x = 150$$

$$5x = 150$$

$$x = 30$$

$$4x = 120$$

The family spent \$120 on food items. **(4 marks)**

- b. Let x be the larger number. Therefore, $x - 8$ will be the smaller number.

$$\therefore 3x = 4(x - 8)$$

$$3x = 4x - 32$$

$$3x - 4x = 4x - 32 - 4x$$

$$-1x = -32$$

$$\frac{-1x}{-1} = \frac{-32}{-1}$$

$$x = 32$$

The larger number is 32, and the smaller number is $32 - 8 = 24$. **(4 marks)**

- c. Let n be the number of hours Monica babysits. Therefore, $n + 7$ will be the number of hours she worked part-time.

$$\therefore 7.5(n + 7) + 4(n) = 110$$

$$7.5n + 52.5 + 4n = 110$$

$$11.5n + 52.5 - 52.5 = 110 - 52.5$$

$$11.5n = 57.5$$

$$\frac{11.5n}{11.5} = \frac{57.5}{11.5}$$

$$n = 5$$

Monica babysat for 5 h and worked part-time for $5 + 7 = 12$ h. This means she worked a total number of 17 h. **(4 marks)**

- d. Let n be the number of nickels. Therefore, $n - 20$ is the number of dimes.

$$\therefore 10(n - 20) + 5n = 550$$

$$10n - 200 + 5n = 550$$

$$15n - 200 = 550$$

$$15n - 200 + 200 = 550 + 200$$

$$15n = 750$$

$$\frac{15n}{15} = \frac{750}{15}$$

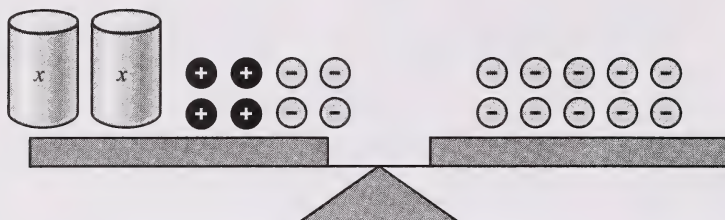
$$n = 50$$

Martin has 50 nickels. **(4 marks)**

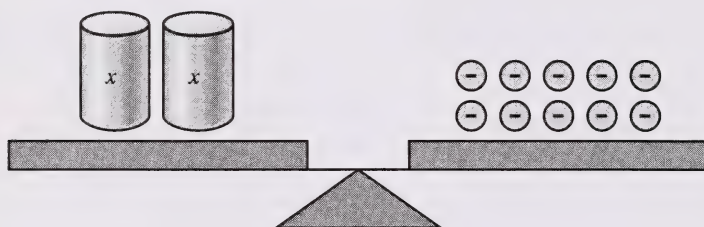
Final Module Assignment Answer Key (30 marks)

1. a. $2x + 4 = -6$ (1 mark)

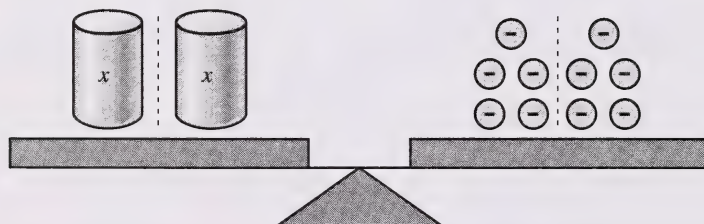
b. Isolate $2x$ by adding -4 to both sides.



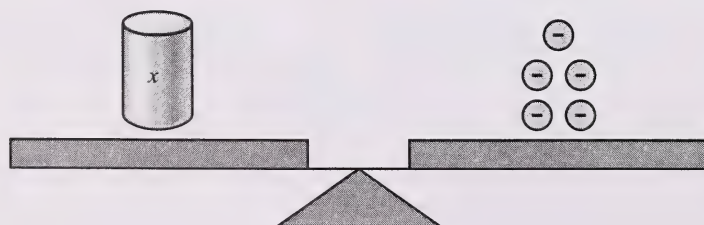
Simplify by removing any zero pairs. This is the result.



Isolate x by dividing each side into two groups.



Solve the equation by keeping one of the groups from each side of the equation.



$\therefore x = 5$ (3 marks)

LS	RS
$5x - 17$	$2x + 4$
$= 5(7) - 17$	$= 2(7) + 4$
$= 35 - 17$	$= 14 + 4$
$= 18$	$= 18$
LS	RS

(1 mark)

3. a. $5(x+2) = 14$
 $5x + 10 = 14$
 $5x + 10 - 10 = 14 - 10$
 $5x = 4$
 $\frac{5x}{5} = \frac{4}{5}$
 $x = \frac{4}{5}$ or $x = 0.8$

(2 marks)

b. $2x - 3 = 12 - x$
 $2x - 3 + 3 = 12 - x + 3$
 $2x = 15 - x$
 $2x + x = 15 - x + x$
 $3x = 15$
 $\frac{3x}{3} = \frac{15}{3}$
 $x = 5$

(2 marks)

c. $\frac{x}{2} + 7 = \frac{x}{3}$
 $3\left(\frac{x}{2}\right) + 6(7) = 3\left(\frac{x}{3}\right)$
 $3x + 42 = 2x$
 $3x + 42 - 42 = 2x - 42$
 $3x = 2x - 42$
 $3x - 2x = 2x - 42 - 2x$
 $x = -42$

(2 marks)

Check

LS	RS
$5(x+2)$	14
$= 5\left(\frac{4}{5} + 2\right)$	
$= 4 + 10$	
$= 14$	
LS	RS

Check

LS	RS
$2x - 3$	$12 - x$
$= 2(5) - 3$	$= 12 - 5$
$= 10 - 3$	$= 7$
$= 7$	
LS	RS

Check

LS	RS
$\frac{x}{2} + 7$	$\frac{x}{3}$
$= \frac{-42}{2} + 7$	$= \frac{-42}{3}$
$= -21 + 7$	$= -14$
$= -14$	
LS	RS

4. The value of the games themselves must be at least \$89.95 (since the maximum cost of shipping and handling that can apply is \$10.00), but cannot be more than \$99.95. Since the cost of the merchandise is in the range of \$65.01 to \$100.00, the cost of shipping and handling is \$10.00.

Total cost will equal the cost of the 5 games plus the shipping.

Let p be the price (in dollars) of one game.

$$\therefore \text{Total cost} = 5p + 10$$

$$99.95 = 5p + 10$$

$$99.95 - 10 = 5p + 10 - 10$$

$$89.95 = 5p$$

$$\frac{89.95}{5} = \frac{5p}{5}$$

$$17.99 = p$$

$$p = 17.99$$

The cost of one game is \$17.99. (3 marks)

5. a. Calculate the area of the two congruent triangles (corresponding to the front and back) and the two rectangles (corresponding to the sides).

First, find the area of each triangle.

$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2}(1.6)(0.6) \\ &= 0.48 \end{aligned}$$

The area of each triangle is 0.48 m^2 . Together, the area of both triangles is $2 \times 0.48 = 0.96 \text{ m}^2$.

Next, find the area of the rectangles.

$$\begin{aligned} A &= \ell w \\ &= 2.0(1.0) \\ &= 2.0 \end{aligned}$$

The area of each rectangle is 2.0 m^2 . Together, the area of both rectangles is $2 \times 2.0 = 4.0 \text{ m}^2$.

$$\begin{aligned} \therefore S &= 0.96 + 4.0 \\ &= 4.96 \end{aligned}$$

The surface area of the tent, with no material on the bottom, is 4.96 m^2 . (4 marks)

b. $V = BH$

$$= \left(\frac{1}{2}bh \right)(2.0)$$

$$= \frac{1}{2}(1.6)(0.6)(2.0)$$

$$= 0.96$$

The volume of the tent is 0.96 m^3 . (2 marks)

6. The wasted material will be the difference between the volume of the rectangular prism of wood and the finished cylinder.

Find the volume of the rectangular prism.

$$V = BH$$

$$= (\ell w)H$$

$$= (100)(1.2)(1.2)$$

$$= 144 \text{ cm}^3$$

Find the volume of cylindrical dowel.

$$V = BH$$

$$= (\pi r^2)H$$

$$= \pi(0.5)^2(100)$$

$$\doteq 78.54 \text{ cm}^3$$

$$\text{Wasted material} \doteq 144 - 78.54$$

$$\doteq 65.5$$

Approximately 65.5 cm^3 of wood is wasted. (4 marks)

7. $V = \frac{1}{3}BH$

$$5.4 = \frac{1}{3}(8.1)H$$

$$3(5.4) = \cancel{3} \left[\frac{1}{\cancel{3}}(8.1)H \right]$$

$$16.2 = 8.1H$$

$$\frac{16.2}{8.1} = \frac{8.1H}{8.1}$$

$$2 = H$$

$$H = 2$$

The pile of sand is 2 m high. (2 marks)

8. Let x be the number of minutes of phone calls to the United States. Therefore, $5x$ is the number of minutes of phone calls within Canada, since the calls are the same length.

Cost = minutes to U.S. $\times$ rate to U.S. + minutes within Canada $\times$ rate within Canada

$$20.75 = (x)(0.23) + (5x)(0.12)$$

$$20.75 = 0.23x + 0.6x$$

$$20.75 = 0.83x$$

$$\frac{20.75}{0.83} = \frac{0.83x}{0.83}$$

$$25 = x$$

$$x = 25$$

Lisa must talk for more than 25 minutes to people in the United States and 125 minutes to people within Canada to make the rate worthwhile. (4 marks)

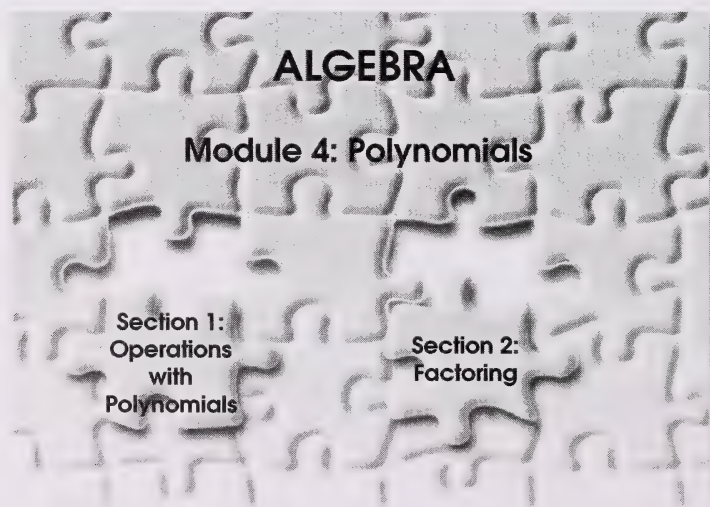
Module 4: Polynomials

Overview

This module begins by introducing some of the terminology of polynomials. The students then become familiar with how to model polynomials using algebra tiles. They look at various ways of representing zero, defining like terms, and examining how to combine like terms using algebra tiles. Evaluating polynomials completes the first activity. The remainder of Section 1 is devoted to performing computations with various kinds of polynomials. Students use algebra tiles to manipulate the polynomials so they can understand the paper-and-pencil computations of adding, subtracting, multiplying, and dividing polynomials.

Section 2 introduces the students to factoring. They review what factoring means with whole numbers and then apply these concepts to algebraic terms and polynomials. Factoring includes finding the common factors and factoring out the greatest common factor (GCF) of a number of terms and polynomials. Common binomial factors and some simple trinomials are also examined.

This module gives students a good understanding of the basics of polynomial algebra. They will be able to carry these concepts forward and build upon them as they further their mathematical experiences in future mathematics courses.



Assessment

The assessment of this module will be based on three assignments:

Assignment Booklet 4A	
Section 1 Assignment	40 marks
Assignment Booklet 4B	
Section 2 Assignment	20 marks
Final Module Assignment	40 marks
TOTAL	100 marks

Section 1: Operations with Polynomials

Key Concepts

- the terminology of polynomials
- modelling polynomials
- evaluating polynomials
- adding and subtracting polynomials
- multiplying binomials
- multiplying monomials and multiplying polynomials by monomials
- dividing polynomials by monomials
- using algebra tiles to justify the algebraic process for computation with polynomials

The basic goals of this section are to ensure that students are able to

- use the basic vocabulary of polynomials
- model various polynomials and use the models to manipulate polynomials
- evaluate polynomials
- add and subtract polynomials of any type
- multiply binomials
- multiply any type of polynomial by a monomial
- divide a polynomial by a monomial
- explain how algebra tiles can be used to justify the algebraic process for computation with polynomials

Section 1: Assignment Answer Key (40 marks)

1. Answers will vary. Sample answers are given.

a. $3x^3 - 2$ (1 mark)

b. $x^2 + 5x + 7$ (1 mark)

2. $\{2a^3b, -a^3b\}, \{-4a, 2a, a\}, \left\{14, -9, \frac{3}{2}\right\}, \{6ab, ab\}, \left\{\frac{5}{2}ab^3, 6ab^3\right\}$ (2 marks)

3. a. The expression is of degree 4. (1 mark)

b. The expression is of degree 3. (1 mark)

4. a. $2x^2 - 3x - 3$ (1 mark)

b. $-x + 10$ (1 mark)

5. a. $3a^2b - 2ab^2 - ab^2 + 6 = 3a^2b - 3ab^2 + 6$

If $a = -3$ and $b = 2$, then

$$\begin{aligned} 3a^2b - 3ab^2 + 6 &= 3(-3)^2(2) - 3(-3)(2)^2 + 6 \\ &= 3(9)(2) - 3(-3)(4) + 6 \\ &= 54 + 36 + 6 \\ &= 96 \quad \text{(2 marks)} \end{aligned}$$

b. $2a^2 - 6a + 4 + 2a = 2a^2 - 4a + 4$

If $a = -3$, then

$$\begin{aligned} 2a^2 - 4a + 4 &= 2(-3)^2 - 4(-3) + 4 \\ &= 2(9) + 12 + 4 \\ &= 18 + 12 + 4 \\ &= 34 \quad \text{(2 marks)} \end{aligned}$$

6. a. One x^2 -tile and two $-x$ -tiles need to be added. (1 mark)

b. Two $-x^2$ -tiles, one x -tile, and three -1 -tiles need to be added. (1 mark)

7. a. $(3x^2 - 4x + 2) + (x^2 + 2x + 6) = 3x^2 - 4x + 2 + x^2 + 2x + 6$
 $= 3x^2 + x^2 - 4x + 2x + 2 + 6$
 $= 4x^2 - 2x + 8 \quad \text{(2 marks)}$

b.
$$\begin{array}{r} 3a^2b + 2ab - 5b - 1 \\ + (6a^2b + 3ab - b + 6) \\ \hline \end{array} \Rightarrow \begin{array}{r} 3a^2b + 2ab - 5b - 1 \\ + (6a^2b + 3ab - b + 6) \\ \hline 9a^2b + 5ab - 6b + 5 \end{array} \quad \text{(2 marks)}$$

c. $(-4y^2 + 2y - 4) - (-6y^2 + 3y - 2) = -4y^2 + 2y - 4 + 6y^2 - 3y + 2$
 $= -4y^2 + 6y^2 + 2y - 3y - 4 + 2$
 $= 2y^2 - y - 2 \quad \text{(2 marks)}$

d.
$$\begin{array}{r} 6a^2 - 4a + 3 \\ - (-4a^2 - 8a - 7) \\ \hline \end{array} \Rightarrow \begin{array}{r} 6a^2 - 4a + 3 \\ + (4a^2 + 8a + 7) \\ \hline 10a^2 + 4a + 10 \end{array} \quad \text{(2 marks)}$$

8. a. $(2xy)(-4x^2) = (2)(-4)(x)(x^2)(y)$
 $= -8x^3y \quad \text{(2 marks)}$

b. $-4x^2(2x^2 + x - 6) = (-4x^2)(2x^2) + (-4x^2)(x) + (-4x^2)(-6)$
 $= (-4)(2)(x^2)(x^2) + (-4)(x^2)(x) + (-4)(-6)(x^2)$
 $= -8x^4 - 4x^3 + 24x^2 \quad \text{(2 marks)}$

c. $(x-6)(x+1) = x(x) + x(1) + (-6)(x) + (-6)(1)$
 $= x^2 + x - 6x - 6$
 $= x^2 - 5x - 6 \quad \text{(2 marks)}$

d. $(x-7)(x+7) = x(x) + x(7) + (-7)(x) + (-7)(7)$
 $= x^2 + 7x - 7x - 49$
 $= x^2 - 49 \quad \text{(2 marks)}$

$$\begin{aligned}
 \text{e. } (x+5)^2 &= (x+5)(x+5) \\
 &= x(x) + x(5) + 5(x) + 5(5) \\
 &= x^2 + 5x + 5x + 25 \\
 &= x^2 + 10x + 25 \quad (2 \text{ marks})
 \end{aligned}$$

$$\begin{aligned}
 \text{9. a. } \frac{24x^2 - 8x}{4x} &= \frac{24x^2}{4x} + \frac{-8x}{4x} \\
 &= 6x - 2 \quad (2 \text{ marks})
 \end{aligned}$$

$$\begin{aligned}
 \text{b. } (25x^4 - 10x^3 + 40x) \div 5x &= \frac{25x^4}{5x} + \frac{-10x^3}{5x} + \frac{40x}{5x} \\
 &= 5x^3 - 2x^2 + 8 \quad (2 \text{ marks})
 \end{aligned}$$

10. Each term was not divided by 5. The student should have divided $15x^2$ by 5 as well.

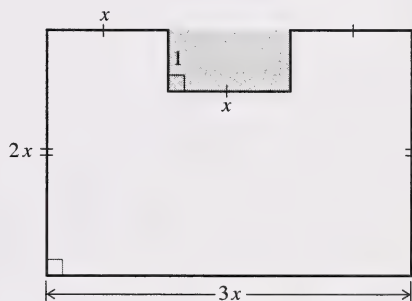
The correct solution is as follows:

$$\begin{aligned}
 \frac{15x^2 + 5}{5} &= \frac{15x^2}{5} + \frac{5}{5} \\
 &= 3x^2 + 1 \quad (2 \text{ marks})
 \end{aligned}$$

11. Method 1

Subtract the area of the shaded rectangle from the area of the large rectangle.

$$\begin{aligned}
 A &= (2x)(3x) - (1)(x) \\
 &= (2)(3)(x)(x) - x \\
 &= 6x^2 - x
 \end{aligned}$$

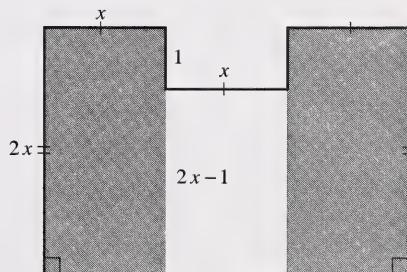


Method 2

Divide the figure into three rectangles and add their areas.

$$\begin{aligned}
 A &= (2x)(x) + (x)(2x-1) + (2x)(x) \\
 &= 2x^2 + 2x^2 - x + 2x^2 \\
 &= 6x^2 - x
 \end{aligned}$$

(2 marks)



Section 2: Factoring

Key Concepts

- understanding what factors are and what factoring involves
- using algebra tiles to model the algebraic process of factoring
- common factor
- factoring trinomials and differences of squares

The basic goals of this section are to ensure that students are able to

- explain what it means to factor a polynomial
- find the greatest common factor (GCF)
- factor polynomials by finding a common factor
- factor trinomials of the form $x^2 + bx + c$
- factor polynomials of the form $A^2 - B^2$, where A and B are both monomial expressions

Section 2: Assignment Answer Key (20 marks)

1. a. $25x^2 = 5 \times 5 \times x \times x$
 $-5 = -1 \times 5$

The GCF is 5. (1 mark)

b. $ab^2 = a \times b \times b$
 $2ab = 2 \times a \times b$
 $-a = -1 \times a$

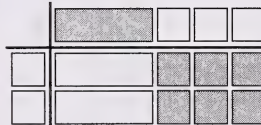
The GCF is a . (1 mark)

2. a. $7x^2y - 14xy = 7xy(x - 2)$ (1 mark)

b. $2x^3 - 6x^2 + 4x = 2x(x^2 - 3x + 2)$ (2 marks)



or



(1 mark)

b. $-2x + 6 = 2(-x + 3)$ or $-2x + 6 = -2(x - 3)$ (1 mark)

4. It is not completely factored because there is still a common factor of 6 between $6x$ and 12. (1 mark)

5. Answers will vary. Sample answers are given.

$$5x + 7 \quad 8y + 3 \quad 2a - 5 \quad -9a + 2$$

(1 mark)

6. $A = \frac{ah}{2} + \frac{bh}{2}$
 $= \frac{1}{2}h(a + b)$ (2 marks)

7. a. $2x^2y - xy = xy(2x - 1)$ (1 mark)

b. $x^2 - 7x + 12 = (x - 4)(x - 3)$ (2 marks)

c. $x^2 + 4x - 12 = (x + 6)(x - 2)$ (2 marks)

d. $3x^2 + 6x + 3 = 3(x^2 + 2x + 1)$
 $= 3(x + 1)(x + 1)$
 $= 3(x + 1)^2$ (2 marks)

8. a. $3x(x - 4) + 2(x - 4) = (x - 4)(3x + 2)$ (1 mark)

b. $x(x + 5) - 6(x + 5) = (x + 5)(x - 6)$ (1 mark)

Final Module Assignment Answer Key (40 marks)

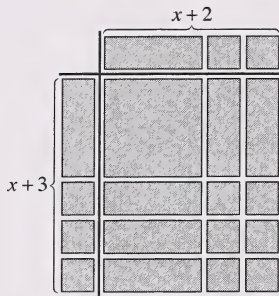
1. The polynomial is of degree 2 and has three terms. Therefore, $-3x^2 + 2x + 1$ is a second-degree trinomial. (2 marks)

2. $m^2 + mn + n^2 + 3m^2 - mn + 6n^2 = 4m^2 + 7n^2$

If $m = -1$ and $n = 2$, then

$$\begin{aligned} 4m^2 + 7n^2 &= 4(-1)^2 + 7(2)^2 \\ &= 4(1) + 7(4) \\ &= 4 + 28 \\ &= 32 \quad (2 \text{ marks}) \end{aligned}$$

3.



$$\therefore x^2 + 5x + 6 = (x + 3)(x + 2) \quad (2 \text{ marks})$$

$$\begin{aligned} 4. \quad \mathbf{a.} \quad (3x^2 - 4x + 5) - (x^2 + 2x + 4) + (2x^2 - 10) &= 3x^2 - 4x + 5 + (-x^2 - 2x - 4) + 2x^2 - 10 \\ &= 3x^2 - 4x + 5 - x^2 - 2x - 4 + 2x^2 - 10 \\ &= 3x^2 - x^2 + 2x^2 - 4x - 2x + 5 - 4 - 10 \\ &= 4x^2 - 6x - 9 \quad \mathbf{(2 \text{ marks})} \end{aligned}$$

b.
$$\begin{array}{r} -6y^2 + 2y - 5 \\ - (y^2 + 7) \\ \hline \end{array} \Rightarrow \begin{array}{r} -6y^2 + 2y - 5 \\ + (-y^2 - 7) \\ \hline -7y^2 + 2y - 12 \end{array} \quad \text{(2 marks)}$$

$$\begin{aligned} \text{c. } 2xy^2(x-y+4) &= 2xy^2(x) + 2xy^2(-y) + 2xy^2(4) \\ &= 2x^2y^2 - 2xy^3 + 8xy^2 \quad (1 \text{ mark}) \end{aligned}$$

d.
$$\frac{27x^2y - 3xy + 9xy^3}{3xy} = \frac{27x^2y}{3xy} - \frac{3xy}{3xy} + \frac{9xy^3}{3xy}$$
$$= 9x - 1 + 3y^2 \quad (2 \text{ marks})$$

$$\begin{aligned} \text{e. } (4x-3)(x-2) &= 4x(x) + 4x(-2) + (-3)(x) + (-3)(-2) \\ &= 4x^2 - 8x - 3x + 6 \\ &= 4x^2 - 11x + 6 \quad \textbf{(2 marks)} \end{aligned}$$

$$\begin{aligned}
 5. \quad A &= \frac{1}{2}bh \\
 &= \frac{1}{2}(4x+6)(2x+3) \\
 &= \left(\frac{4x+6}{2}\right)(2x+3) \\
 &= (2x+3)(2x+3) \\
 &= 2x(2x) + 2x(3) + 3(2x) + 3(3) \\
 &= 4x^2 + 6x + 6x + 9 \\
 &= 4x^2 + 12x + 9 \quad \text{(3 marks)}
 \end{aligned}$$

$$\begin{aligned}
 6. \quad a. \quad P &= 4(4x) + 5x + 4x + 2(6x) \\
 &= 16x + 9x + 12x \\
 &= 37x
 \end{aligned}$$

The amount of fencing required is $37x$ units. (2 marks)

$$\begin{aligned}
 b. \quad A &= (4x)(6x) \\
 &= 24x^2
 \end{aligned}$$

The area of the field is $24x^2$ square units. (1 mark)

$$\begin{aligned}
 7. \quad 8ab^2c^3 &= 2 \times 2 \times 2 \times a \times b \times b \times c \times c \times c \\
 -16a^3bc &= -1 \times 2 \times 2 \times 2 \times a \times a \times a \times b \times c \\
 12a^2b^4 &= 2 \times 2 \times 3 \times a \times a \times b \times b \times b \times b
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{GCF} &= 2 \times 2 \times a \times b \\
 &= 4ab \quad \text{(2 marks)}
 \end{aligned}$$

$$\begin{aligned}
 8. \quad a. \quad x^2 - 8x + 16 &= (x-4)(x-4) \\
 &= (x-4)^2 \quad \text{(2 marks)}
 \end{aligned}$$

$$b. \quad 6xy^2 + 3xy = 3xy(2y+1) \quad \text{(1 mark)}$$

$$\begin{aligned}
 c. \quad 5x^2y - 125y &= 5y(x^2 - 25) \\
 &= 5y(x-5)(x+5) \quad \text{(2 marks)}
 \end{aligned}$$

$$\begin{aligned}
 d. \quad -2x^2 + 10x + 12 &= -2(x^2 - 5x - 6) \\
 &= -2(x-6)(x+1) \quad \text{(2 marks)}
 \end{aligned}$$

9. Answers will vary. Sample answers are given.

$$\begin{aligned}
 x^2 + 13x + 12 &= (x+1)(x+12) & x^2 - 13x + 12 &= (x-1)(x-12) \\
 x^2 + 8x + 12 &= (x+2)(x+6) & x^2 - 8x + 12 &= (x-2)(x-6) \\
 x^2 + 7x + 12 &= (x+3)(x+4) & x^2 - 7x + 12 &= (x-3)(x-4)
 \end{aligned}$$

(3 marks)

10. It cannot be factored over the integers because there are no two numbers whose product is 4 and whose sum is 2. (2 marks)

11. $x^2 + 4x + 4 = (x+2)(x+2)$ or $(x+2)^2$ (1 mark)

12. a. Christine and Michael are modelling the following addition question:

$$(2x^2 - 2x + 3) + (-x^2 + 2x + 2) \quad (2 \text{ marks})$$

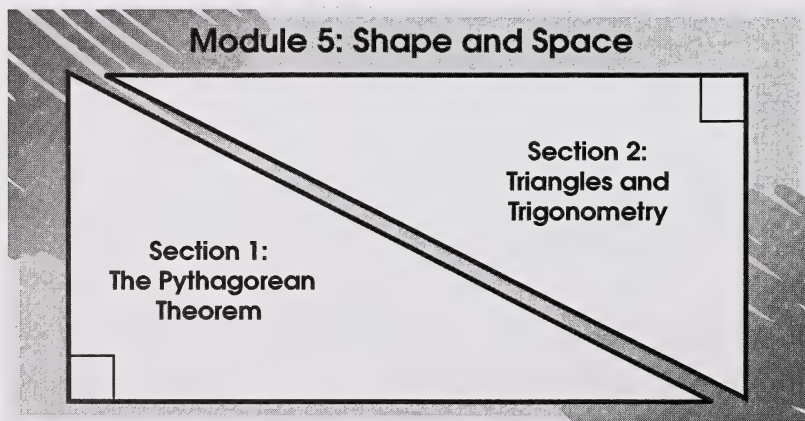
b. To obtain the answer to the addition questions, the students should collect similar terms and remove zero pairs. The answer is $x^2 + 5$. (2 marks)

Module 5: Shape and Space

Overview

Section 1 of this module focuses on skills necessary for the application of the Pythagorean Theorem. Activity 1 reintroduces the student to squares and square roots. Students distinguish between principal and negative roots, review methods for estimating roots, and use calculators to obtain approximations. These approaches are extended to include cubes and cube roots. In Activity 2, students solve simple equations involving squares and use these equations to model practical problems. Depending on the context, students select either principal roots, or negative roots, or both for the solution sets of these equations. In Activity 3, students review the details of the Pythagorean Theorem, its notation and terminology, and its application.

Section 2 focuses on the triangle. Specifically, students review angles and their classification, congruent and similar triangles, and elementary right-triangle trigonometry. The students then apply the discussion of similar triangles to similar figures, scale diagrams, and the effects of scaling. Throughout the students' exploration, practical problems and everyday contexts reinforce concepts and processes.



Assessment

The assessment of this module will be based on three assignments:

Assignment Booklet 5A	
Section 1 Assignment	20 marks
Assignment Booklet 5B	
Section 2 Assignment	40 marks
Final Module Assignment	40 marks
TOTAL	100 marks

Section 1: The Pythagorean Theorem

Key Concepts

- perfect squares and perfect cubes
- square roots and cube roots
- principal roots and negative roots
- equations containing squares
- the Pythagorean Theorem

The basic goals of this section are to ensure that students are able to

- distinguish between exact values and decimal approximations of square roots and cube roots
- differentiate between principal square roots and negative square roots
- give examples where answers would involve the principal square root, the negative square root, or both
- document and explain the calculator key sequence when calculating a square root or a cube root
- use the Pythagorean Theorem to solve problems in two dimensions

Section 1: Assignment Answer Key (20 marks)

- $-\sqrt{144} = -12$ (1 mark)
 - $$\begin{aligned} \sqrt{\frac{16}{81}} &= \frac{\sqrt{16}}{\sqrt{81}} \\ &= \frac{4}{9} \quad (1 \text{ mark}) \end{aligned}$$
 - $$\begin{aligned} \sqrt[3]{1331} &= \sqrt[3]{11 \times 11 \times 11} \\ &= 11 \quad (1 \text{ mark}) \end{aligned}$$
- Because $\sqrt{100} = 10$ and $\sqrt{121} = 11$, $\sqrt{110}$ is between 10 and 11. Because 110 is almost halfway between 100 and 121, $\sqrt{110}$ is about 10.5. (2 marks)
- Answers will vary. Some examples are given.
1, 4, 9, 16, 25, 36, 49, 64, 81, 100, ... (1 mark)
- $$\begin{aligned} 6^2 + 8^2 &= x^2 \\ 36 + 64 &= x^2 \\ 100 &= x^2 \\ x^2 &= 100 \\ x &= \pm\sqrt{100} \\ x &= \pm 10 \end{aligned}$$

Because the length of a side is always positive, $x = 10$. (2 marks)

$$\begin{aligned}
 \text{b.} \quad & y^2 + 12^2 = 20^2 \\
 & y^2 + 144 = 400 \\
 & y^2 + 144 - 144 = 400 - 144 \\
 & y^2 = 256 \\
 & y = \pm\sqrt{256} \\
 & y = \pm 16
 \end{aligned}$$

Because length of a side is always positive, $y = 16$. (2 marks)

5. Let w represent the length of the guy wire.

$$\begin{aligned}
 \therefore 4^2 + 10^2 &= w^2 \\
 16 + 100 &= w^2 \\
 \sqrt{116} &= w \\
 w &= \sqrt{116} \\
 &\approx 10.8
 \end{aligned}$$

The total length of wire required is about $10.8 + 2 = 12.8$ m. (2 marks)

$$\begin{aligned}
 \text{6. a.} \quad & x^2 = 0.81 \\
 & x = \pm\sqrt{0.81} \\
 & = \pm 0.9 \quad (1 \text{ mark})
 \end{aligned}$$

$$\begin{aligned}
 \text{b.} \quad & 4x^2 - 17 = 83 \\
 & 4x^2 - 17 + 17 = 83 + 17 \\
 & 4x^2 = 100 \\
 & \frac{4x^2}{4} = \frac{100}{4} \\
 & x^2 = 25 \\
 & x = \pm\sqrt{25} \\
 & = \pm 5 \quad (2 \text{ marks})
 \end{aligned}$$

$$\begin{aligned}
 \text{c.} \quad & \frac{x^2}{2} = \frac{x^2}{6} + 27 \\
 & \frac{3}{1} \left(\frac{x^2}{3} \right) = \frac{1}{1} \left(\frac{x^2}{1} \right) + 6(27) \\
 & 3x^2 = x^2 + 162 \\
 & 3x^2 - x^2 = x^2 + 162 - x^2 \\
 & 2x^2 = 162 \\
 & \frac{2x^2}{2} = \frac{162}{2} \\
 & x^2 = 81 \\
 & x = \pm\sqrt{81} \\
 & = \pm 9 \quad (3 \text{ marks})
 \end{aligned}$$

7. You cannot take the square root of a negative number because you need two numbers that are the same to determine a square root. If you take a positive number multiplied by itself, the product is positive. If you take a negative number multiplied by itself, the product is also positive. The result will not be negative. **(2 marks)**

Section 2: Triangles and Trigonometry

Key Concepts

- acute, obtuse, right, straight, and reflex angles
- congruent and similar triangles
- trigonometry
- sine, cosine, and tangent
- enlargements and reductions (dilatations)
- scale factor

The basic goals of this section are to ensure that students are able to

- estimate, measure, and classify angles as acute, obtuse, right, straight, and reflex
- recognize when two triangles are similar and use the properties of similar triangles to solve triangles
- explain the meaning of sine, cosine, and tangent ratios in right angles
- use trigonometry to calculate an unknown side or an unknown angle in a right triangle
- model and solve problem situations involving one right triangle
- use ordered pairs to draw designs in all four quadrants of the coordinate grid
- draw and interpret scale diagrams involving enlargements and reductions

Section 2: Assignment Answer Key (40 marks)

$$\begin{aligned}
 1. \quad \angle A &= 180^\circ - (84^\circ + 42^\circ) & \angle E &= 54^\circ \\
 &= 180^\circ - 126^\circ \\
 &= 54^\circ \\
 \angle B &= 84^\circ & \angle D &= 180^\circ - (54^\circ + 42^\circ) \\
 & & &= 180^\circ - 96^\circ \\
 & & &= 84^\circ \\
 \angle C &= 42^\circ & \angle F &= 42^\circ
 \end{aligned}$$

Because the corresponding angles are equal, $\triangle ABC$ is similar to $\triangle EDF$. **(2 marks)**

2. a. Use similar triangles to find the value of r .

$$\begin{aligned}
 \frac{r}{5} &= \frac{3}{10} \\
 \frac{1}{5} \left(\frac{r}{1} \right) &= \frac{1}{10} \left(\frac{3}{2} \right) \\
 r &= \frac{3}{2} \\
 &= 1.5 \text{ cm} \quad \textbf{(2 marks)}
 \end{aligned}$$

- b. Use the Pythagorean Theorem to find the slant height, s .

$$s^2 = 3^2 + 10^2$$

$$s^2 = 9 + 100$$

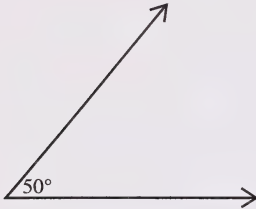
$$s^2 = 109$$

$$s = \sqrt{109}$$

$$\approx 10.4$$

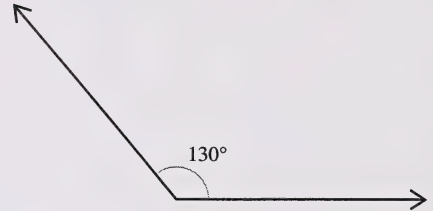
The slant height is about 10.4 cm. (2 marks)

3. a.



This is an acute angle. (2 marks)

b.



This is an obtuse angle. (2 marks)

4. Draw a diagram representing the situation. Let y represent the height of the tree.

Find the value of x using similar triangles.

$$\frac{x}{1} = \frac{x+3}{2}$$

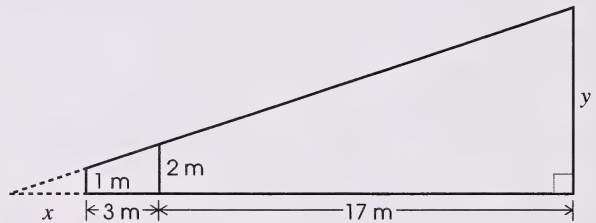
$$x = \frac{x+3}{2}$$

$$2(x) = 1\left(\frac{x+3}{2}\right)$$

$$2x = x+3$$

$$2x - x = x + 3 - x$$

$$x = 3$$



Find the value of y using similar triangles.

$$\frac{y}{3+3+17} = \frac{1}{3}$$

$$\frac{y}{23} = \frac{1}{3}$$

$$23\left(\frac{y}{23}\right) = 23\left(\frac{1}{3}\right)$$

$$y = \frac{23}{3}$$

$$\approx 7.7$$

The height of the tree is about 7.7 m. (4 marks)

5. Because $\angle PSR$ and $\angle QST$ are vertically opposite angles, $\angle PSR = \angle QST$. So, because $\angle PSR = \angle QST$, $PS = ST$, and $RS = QS$, $\triangle PSR \cong \triangle TSQ$ by SAS. (2 marks)

6. a. $\sin A = 0.2233$

$\angle A \doteq 13^\circ$ (1 mark)

b. $\tan A = 1.6$

$\angle A \doteq 58^\circ$ (1 mark)

7. a. $\sin D = \frac{EF}{DF}$ or $\sin D = \frac{EG}{DE}$ (2 marks)

b. $\cos F = \frac{EF}{DF}$ or $\cos F = \frac{FG}{EF}$ (2 marks)

8. a. $\cos B = \frac{\text{adj}}{\text{hyp}}$

$\cos B = \frac{6}{10}$

$\cos B = 0.6$

$\angle B \doteq 53^\circ$ (2 marks)

b. $6^2 + b^2 = 10^2$

$36 + b^2 = 100$

$36 + b^2 - 36 = 100 - 36$

$b^2 = 64$

$b = \sqrt{64}$

$= 8 \text{ cm}$ (2 marks)

9. The value of $\cos A$ is always between 0 and 1 because the hypotenuse is always longer than the adjacent side. This means the ratio, $\frac{\text{adjacent}}{\text{hypotenuse}}$, will be less than 1. Also, because both the adjacent side and the hypotenuse are positive, the ratio will be greater than zero. (2 marks)

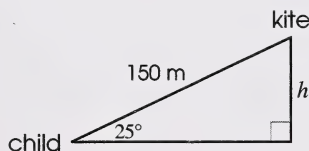
10. $\sin 25^\circ = \frac{\text{opp}}{\text{hyp}}$

$\sin 25^\circ = \frac{h}{150}$

$150(\sin 25^\circ) = 150\left(\frac{h}{150}\right)$

$150 \sin 25^\circ = h$

$h \doteq 63$



The kite is approximately 63 m high. The assumption made is that the ground is the horizontal (as shown in the diagram), not Benny's eye level. (4 marks)

11. To calculate the area of $\triangle ABC$, you need to find the length of AC .

First, find the lengths of AR and CR .

$\tan 37^\circ = \frac{4.2}{AR}$

$AR(\tan 37^\circ) = AR\left(\frac{4.2}{AR}\right)$

$AR \tan 37^\circ = 4.2$

$\frac{AR \tan 37^\circ}{\tan 37^\circ} = \frac{4.2}{\tan 37^\circ}$

$AR = \frac{4.2}{\tan 37^\circ}$

$\tan 59^\circ = \frac{4.2}{CR}$

$CR(\tan 59^\circ) = CR\left(\frac{4.2}{CR}\right)$

$CR \tan 59^\circ = 4.2$

$\frac{CR \tan 59^\circ}{\tan 59^\circ} = \frac{4.2}{\tan 59^\circ}$

$CR = \frac{4.2}{\tan 59^\circ}$

Note to teacher: Accept rounded values.

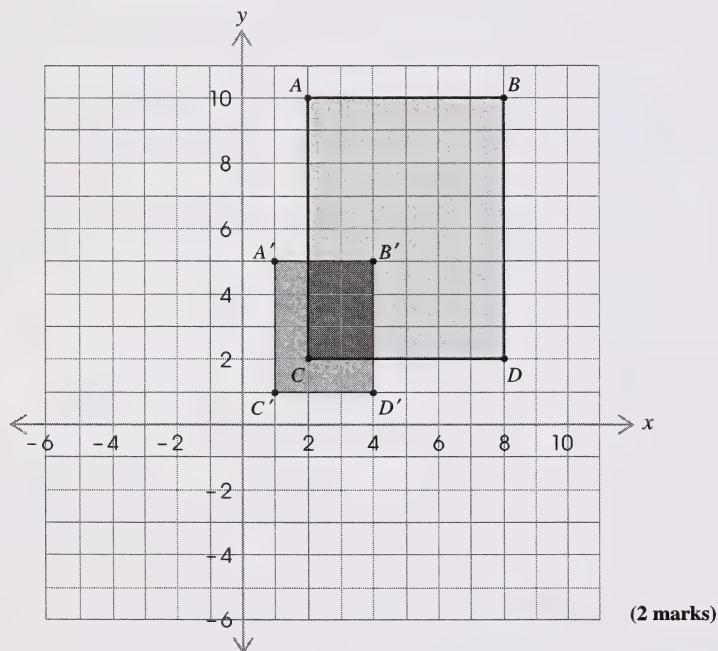
Now, calculate AC .

$$\begin{aligned} AC &= AR + CR \\ &= \frac{4.2}{\tan 37^\circ} + \frac{4.2}{\tan 59^\circ} \\ &\approx 8.10 \text{ cm} \end{aligned}$$

Calculate the area of $\triangle ABC$.

$$\begin{aligned} \text{Area} &= \frac{1}{2}bh \\ &\approx \frac{1}{2}(8.10)(4.2) \\ &\approx 17.0 \text{ cm}^2 \quad (5 \text{ marks}) \end{aligned}$$

12. a.



b. $(x, y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)$ (1 mark)

Final Module Assignment Answer Key (40 marks)

1. $\sqrt{75} \doteq 8.7$ (1 mark)

2. a. $-\sqrt{25} = -5$ (1 mark)

b. $\sqrt{\frac{169}{25}} = \frac{13}{5}$ (1 mark)

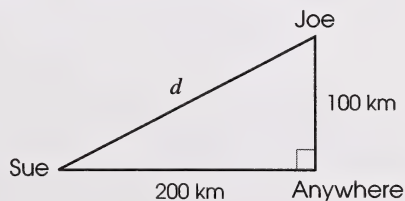
3. a. $x^2 = 625$
 $x = \pm\sqrt{625}$
 $= 25 \text{ or } -25$ (2 marks)

b. $5x^2 - 4 = x^2 + 60$
 $5x^2 - 4 - x^2 = x^2 + 60 - x^2$
 $4x^2 - 4 = 60$
 $4x^2 - 4 + 4 = 60 + 4$
 $4x^2 = 64$
 $\frac{4x^2}{4} = \frac{64}{4}$
 $x^2 = 16$
 $x = \pm\sqrt{16}$
 $= 4 \text{ or } -4$ (2 marks)

c. $2x^2 = 3x^2 + 25$
 $2x^2 - 3x^2 = 3x^2 + 25 - 3x^2$
 $-1x^2 = 25$
 $\frac{-1x^2}{-1} = \frac{25}{-1}$
 $x^2 = -25$

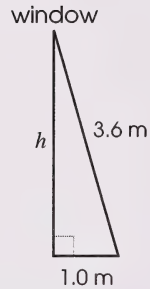
A number squared can never be negative; so, there is no solution. (2 marks)

4. $d^2 = 100^2 + 200^2$
 $d^2 = 10\,000 + 40\,000$
 $d^2 = 50\,000$
 $d = \sqrt{50\,000}$
 $\doteq 224$



Sue and Joe are about 224 km apart. (3 marks)

$$\begin{aligned}
 5. \quad h^2 + 1.0^2 &= 3.6^2 \\
 h^2 + 1 &= 12.96 \\
 h^2 + 1 - 1 &= 12.96 - 1 \\
 h^2 &= 11.96 \\
 h &= \sqrt{11.96} \\
 &\doteq 3.5
 \end{aligned}$$

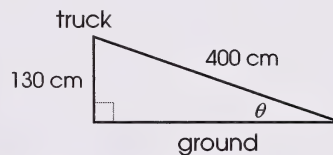


The window is about 3.5 m up the side of the house. (3 marks)

$$\begin{aligned}
 6. \quad \text{a.} \quad \sin A &= \frac{5}{16} \\
 \angle A &\doteq 18^\circ \quad (2 \text{ marks})
 \end{aligned}$$

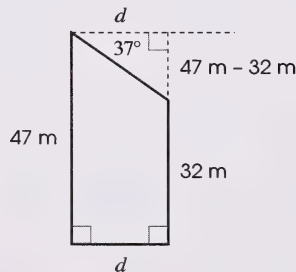
$$\begin{aligned}
 \text{b.} \quad \tan A &= 3.5 \\
 \angle A &= 74^\circ \quad (2 \text{ marks})
 \end{aligned}$$

$$\begin{aligned}
 7. \quad \sin \theta &= \frac{\text{opp}}{\text{hyp}} \\
 &= \frac{130}{400} \\
 \theta &\doteq 19^\circ
 \end{aligned}$$



The ramp makes an angle of about 19° with the ground. (3 marks)

$$\begin{aligned}
 8. \quad \tan 37^\circ &= \frac{\text{opp}}{\text{adj}} \\
 \tan 37^\circ &= \frac{47 - 32}{d} \\
 \tan 37^\circ &= \frac{15}{d} \\
 d(\tan 37^\circ) &= \cancel{d} \left(\frac{15}{\cancel{d}} \right) \\
 d \tan 37^\circ &= 15 \\
 \frac{d \tan 37^\circ}{\tan 37^\circ} &= \frac{15}{\tan 37^\circ} \\
 d &\doteq 20
 \end{aligned}$$



The distance between the two masts is about 20 m. (4 marks)

9. The tangent ratio does not need to lie between 0 and 1 because the opposite side of the angle is not necessarily smaller than the adjacent side. The ratio could be any number greater than 0. **(3 marks)**

10. Find the length of BC .

$$(BC)^2 + 12^2 = 15^2$$

$$(BC)^2 + 144 = 225$$

$$(BC)^2 + 144 - 144 = 225 - 144$$

$$(BC)^2 = 81$$

$$BC = \sqrt{81}$$

$$= 9$$

Since $BC = 9$ and, therefore, the smallest side, $\angle A$, must be the smallest angle.

$$\therefore \cos \angle A = \frac{12}{15}$$

$$\doteq 37^\circ \quad \textbf{(3 marks)}$$

11. a. $\frac{AC}{BC} = \frac{DE}{BE}$

$$\frac{AC}{75} = \frac{56}{40}$$

$$\frac{AC}{75} = \frac{7}{5}$$

$$\cancel{75} \left(\frac{AC}{\cancel{75}} \right) = \cancel{75} \left(\frac{7}{\cancel{5}} \right)$$

$$AC = 105 \quad \textbf{(2 marks)}$$

- b. Answers will vary. A sample answer is given. You would require the measures of $\angle A$ and $\angle C$ because $\angle B = 180^\circ - (\angle A + \angle C)$. **(2 marks)**

12. a. $(x, y) \rightarrow (1.5x, 1.5y)$ **(2 marks)**

b. $B(4, 5) \rightarrow B'(6, 7.5)$

$$C(6, 1) \rightarrow C'(9, 1.5)$$

(2 marks)

Final Test

Security

Included here is the answer key to the Final Test and the student's copy of the Final Test. Teachers should keep these secure against unauthorized student access. Students should not have access to the test until it is assigned in a supervised situation. The answers should be stored securely and retained by the teacher at all times.

MATHEMATICS PREPARATION 10

FINAL TEST ANSWER KEY

Part A: Numbers (40 marks)

1. a. Find the least common multiple (LCM) of 2, 6, and 10.

2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, (30), 32, ...

6, 12, 18, 24, (30), 36, ...

10, 20, (30), 40, ...

The least common multiple is 30. Therefore, the next time all three families buy convenience food on the same day is on June 30. (2 marks)

- b. The number of common multiples of 2 and 6 will be the number of times the Smith family and the Ray family buy convenience food on the same day before all three families buy convenience food.

6, 12, 18, 24, (30), 36, ...

The Smith family and the Ray family will buy convenience food on the same day 4 times. (2 marks)

2. Length of blackboard = $4\frac{7}{8} + 1\frac{1}{2}$

$$= \frac{39}{4} + \frac{3}{2}$$

$$= \frac{39}{4} \times \frac{1}{1} + \frac{3}{2} \times \frac{2}{2}$$

$$= \frac{39}{4} + \frac{3}{2}$$

$$= \frac{39}{4} + \frac{6}{4}$$

$$= 6\frac{1}{2} \text{ m (3 marks)}$$

3. a. $\frac{90}{40} = \frac{9}{4}$

$$= 2\frac{1}{4}$$

An unfit adult's heart rate is $2\frac{1}{4}$ times faster than a trained runner's heart rate. (2 marks)

b. $\frac{9}{4} = \frac{225}{100}$

$$= 225\% \text{ (1 mark)}$$

4. $\sqrt{49} < \sqrt{60} < \sqrt{64}$
 $7 < \sqrt{60} < 8$

Because 60 is closer to 64, $\sqrt{60} \doteq 7.8$.

To check this without using the $(\sqrt{\quad})$ on your calculator, multiply 7.8 by 7.8. This is equal to 60.84. **(3 marks)**

5. Use the Pythagorean Theorem to determine whether $\triangle ABC$ is a right triangle.

LS	RS
$1.5^2 + 2^2$	2.5^2
$= 2.25 + 4$	$= 6.25$
$= 6.25$	
LS	= RS

Therefore, $\triangle ABC$ is a right triangle. **(3 marks)**

6. $(-4)^3 = (-4) \times (-4) \times (-4)$ **(1 mark)**

7. $3^{-4} = \frac{1}{3^4}$ $(-3)^4 = (-3) \times (-3) \times (-3) \times (-3)$
 $= \frac{1}{81}$ $= 81$

Charlie is not correct. **(2 marks)**

8. a. $\frac{5x^2y^3}{x^4y^2} = 5x^{2-4}y^{3-2}$
 $= 5x^{-2}y$
 $= \frac{5y}{x^2}$ **(2 marks)**

b. $(-4x^2y^{-2})(2x) = (-4)(2)(x^{2+1})(y^{-2})$
 $= -8x^3y^{-2}$
 $= \frac{-8x^3}{y^2}$ **(2 marks)**

c. $\frac{(2xy)(3x)}{(15xy)^0} = \frac{6x^2y}{1}$
 $= 6x^2y$ **(2 marks)**

9. The temperature dropped 9°C . The integer, -9 represents this temperature change. (2 marks)
10. Sara started with $+3$ (). She needed to take away five positive tiles, but there weren't enough tiles to take away. So, she had to add two zero pairs ( to the counters to have five positive tiles to take away, thus leaving Sara with 2 negative tiles (). (2 marks)
11. a.
$$\begin{aligned}\frac{2}{3} + \frac{1}{6} + 1\frac{1}{2} - \frac{2}{9} &= \frac{2}{3} + \frac{1}{6} + \frac{3}{2} - \frac{2}{9} \\ &= \frac{2}{3} + \frac{1}{6} + \frac{8}{3} - \frac{2}{9} \\ &= \frac{2}{3} + \frac{1}{9} - \frac{2}{9} \\ &= \frac{6}{9} + \frac{1}{9} - \frac{2}{9} \\ &= \frac{5}{9} \quad (2 \text{ marks})\end{aligned}$$
- b.
$$\begin{aligned}5 - \left[\left(\frac{1}{12} + \frac{2}{3} \right) \div \frac{1}{4} \right] &= 5 - \left[\left(\frac{1}{12} + \frac{8}{12} \right) \div \frac{1}{4} \right] \\ &= 5 - \left(\frac{9}{12} \div \frac{1}{4} \right) \\ &= 5 - \left(\frac{3}{4} \times \frac{4}{1} \right) \\ &= 5 - 3 \\ &= 2 \quad (2 \text{ marks})\end{aligned}$$
- c.
$$\begin{aligned}3^{-2} \times 3^4 + \left[2^2 + \left(\frac{1}{2} \right)^{-3} \right] &= 3^{-2+4} + \left[4 + \left(\frac{2}{1} \right)^3 \right] \\ &= 3^2 + (4 + 8) \\ &= 9 + 12 \\ &= 21 \quad (2 \text{ marks})\end{aligned}$$
12. Answers will vary. Sample answers are given.
- a. The numbers could be 2 and 3 ($2 \div 3 = 0.\bar{6}$). (1 mark)
- b. No, there are many numbers! For example, $\frac{2}{3} = \frac{4}{6} = \frac{8}{12} = \dots$. (1 mark)

13. Determine the cost of the food.

$$\begin{aligned}\text{Cost of food} &= (2 \times 7.50) + (3 \times 6.20) + (4 \times 0.75) + (4 \times 1.50) \\ &= 15.00 + 18.60 + 3.00 + 6.00 \\ &= \$42.60\end{aligned}$$

Determine the amount of tip.

$$\begin{aligned}\text{Tip} &= 15\% \text{ of } 42.60 \\ &= 0.15 \times 42.60 \\ &= \$6.39\end{aligned}$$

$$\begin{aligned}\therefore \text{Money left over} &= 50.00 - (42.60 + 6.39) \\ &= 50.00 - 48.99 \\ &= 1.01\end{aligned}$$

George has \$1.01 left over. (3 marks)

Part B: Patterns and Relations (26 marks)

1. a. Develop a formula to determine the price per yearbook. Let P represent the price per yearbook, and let n represent the number of yearbooks ordered.

$$\begin{aligned}\text{Price per yearbook} &= \frac{\text{total cost of yearbooks}}{\text{number of yearbooks ordered}} \\ P &= \frac{1250 + 7n}{n}\end{aligned}$$

Substitute $n = 250$ into the formula.

$$\begin{aligned}P &= \frac{1250 + 7(250)}{250} \\ &= \frac{1250 + 1750}{250} \\ &= \frac{3000}{250} \\ &= 12\end{aligned}$$

The price per yearbook is \$12. (3 marks)

$$\begin{aligned}
 \text{b.} \quad P &= \frac{1250 + 7n}{n} \\
 9.50 &= \frac{1250 + 7n}{n} \\
 n(9.50) &= n\left(\frac{1250 + 7n}{n}\right) \\
 9.50n &= 1250 + 7n \\
 9.50n - 7n &= 1250 + 7n - 7n \\
 2.50n &= 1250 \\
 \frac{2.50n}{2.50} &= \frac{1250}{2.50} \\
 n &= 500
 \end{aligned}$$

The school must order 500 yearbooks. (2 marks)

2. Let ℓ represent the cost of a large pizza, and let s represent the cost of a small pizza.

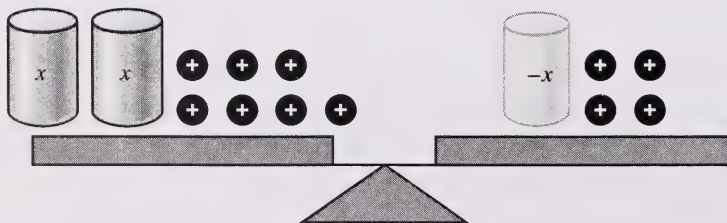
$$\therefore \ell = 2s - 2.50 \quad (1 \text{ mark})$$

$$\begin{aligned}
 \text{3. a.} \quad (3a^3 - 2a^2 + 4) - (3a^3 - a^2 - 6) &= 3a^3 - 2a^2 + 4 + (-3a^3 + a^2 + 6) \\
 &= 3a^3 - 3a^3 - 2a^2 + a^2 + 4 + 6 \\
 &= -a^2 + 10 \quad (2 \text{ marks})
 \end{aligned}$$

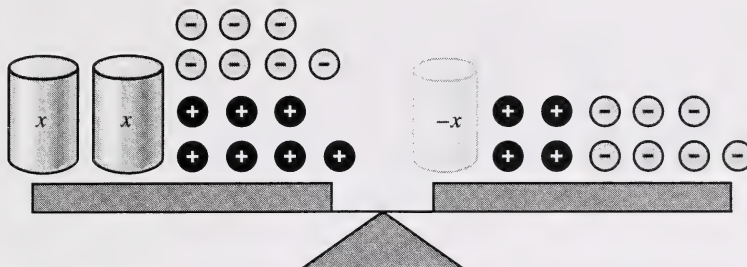
$$\begin{aligned}
 \text{b.} \quad \frac{16n^4 - 8n^2 + 4n}{4n} &= \frac{16n^4}{4n} - \frac{8n^2}{4n} + \frac{4n}{4n} \\
 &= 4n^3 - 2n + 1 \quad (2 \text{ marks})
 \end{aligned}$$

$$\begin{aligned}
 \text{c.} \quad (x+5)(x-2) &= x(x) + x(-2) + 5(x) + 5(-2) \\
 &= x^2 - 2x + 5x - 10 \\
 &= x^2 + 3x - 10 \quad (2 \text{ marks})
 \end{aligned}$$

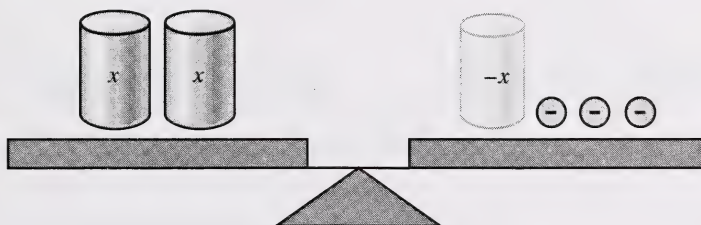
4. Step 1: Model the equation.



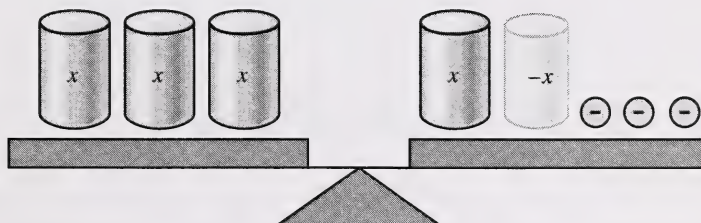
Step 2: Isolate $2x$ on the left side by adding -7 to both sides.



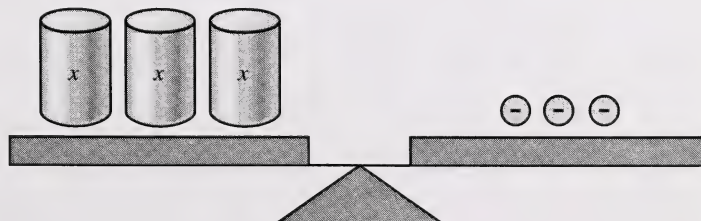
Step 3: Simplify by removing zero pairs. This is the result.



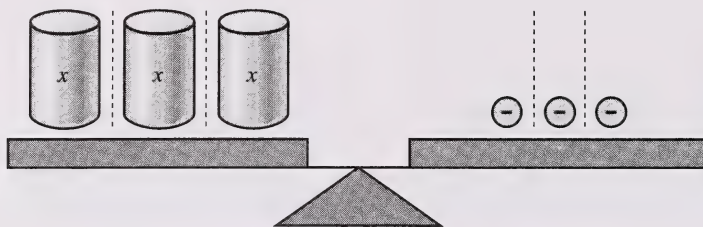
Step 4: Isolate -3 on the right side by adding x to both sides.



Step 5: Simplify by removing zero pairs. This is the result.



Step 6: Isolate x by dividing each side into three groups.



Step 7: Solve the equation by keeping one of the groups from each side of the equation.



$\therefore x = -1$ (5 marks)

5. a. $3x^2y^2 - 6x = 3x(xy^2 - 2)$ (1 mark)

b. $x^2 - 4x - 12 = (x - 6)(x + 2)$ (1 mark)

6. Let b represent the amount (in dollars) Judy made baby-sitting.

Method 1: Using Ratios

$$\begin{aligned}\frac{80}{100} &= \frac{60}{b} \\ \frac{4}{5} &= \frac{60}{b} \\ b\left(\frac{4}{5}\right) &= \cancel{5}\left(\frac{60}{\cancel{5}}\right) \\ \frac{4b}{5} &= 60 \\ \frac{1}{\cancel{4}}\left(\frac{\cancel{4}b}{\cancel{5}}\right) &= \frac{5}{\cancel{4}}\left(\frac{15}{\cancel{5}}\right) \\ b &= 75\end{aligned}$$

Judy made \$75 baby-sitting. (3 marks)

Method 2: Using an Equation

$$\begin{aligned}60 &= 80\% \text{ of } b \\ 60 &= 0.8 \times b \\ \frac{60}{0.8} &= \frac{0.8b}{0.8} \\ 75 &= b \\ b &= 75\end{aligned}$$

$$\begin{aligned}
 7. \quad & 3x + 5 = -x + 41 \\
 & 3x + 5 + x = -x + 41 + x \\
 & 4x + 5 = 41 \\
 & 4x + 5 - 5 = 41 - 5 \\
 & 4x = 36 \\
 & \frac{4x}{4} = \frac{36}{4} \\
 & x = 9
 \end{aligned}$$

(3 marks)

Check

LS	RS
$3x + 5$	$-x + 41$
$= 3(9) + 5$	$= -9 + 41$
$= 27 + 5$	$= 32$
$= 32$	
LS	= RS

$$\begin{aligned}
 8. \quad & V = \pi r^2 h \\
 & \frac{V}{\pi r^2} = \frac{\pi r^2 h}{\pi r^2} \\
 & \frac{V}{\pi r^2} = h \\
 & h = \frac{V}{\pi r^2} \quad (1 \text{ mark})
 \end{aligned}$$

Part C: Shape and Space (23 marks)

1. Draw a net representing the three faces (top, bottom, and side). **Note:** $A_1 = A_2$.

Determine the areas of A_1 and A_2 , the top and bottom faces.

$$\begin{aligned}
 A_1 &= \pi r^2 \\
 &= \pi (5.3)^2 \\
 &\doteq 88.2 \text{ cm}^2 \quad \therefore A_2 \doteq 88.2 \text{ cm}^2
 \end{aligned}$$

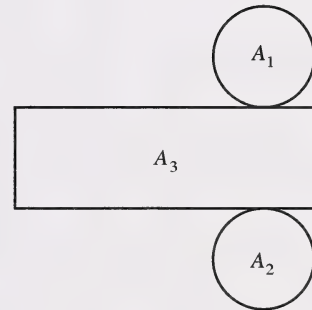
Determine the area of A_3 .

$$\begin{aligned}
 A_3 &= \ell w \\
 &= (\pi d)(18) \\
 &= \pi (10.6)(18) \\
 &\doteq 599.4 \text{ cm}^2
 \end{aligned}$$

Calculate the surface area of the can.

$$\begin{aligned}
 S &= A_1 + A_2 + A_3 \\
 &\doteq 88.2 + 88.2 + 599.4 \\
 &\doteq 775.8
 \end{aligned}$$

The surface area of the can is about 776 cm^2 . (5 marks)



$$\begin{array}{lll}
 \text{2. a. length} = 12 - 2(3) & \text{width} = 10 - 2(3) & \text{height} = 3 \text{ cm} \\
 = 12 - 6 & = 10 - 6 & \\
 = 6 \text{ cm} & = 4 \text{ cm} &
 \end{array}$$

(3 marks)

$$\begin{array}{l}
 \text{b. } V = \ell wh \\
 = (6)(4)(3) \\
 = 72
 \end{array}$$

The volume of the box is 72 cm^3 . (2 marks)

$$\begin{array}{l}
 \text{c. Percent wasted} = \frac{\text{cardboard that was cut out}}{\text{total amount of cardboard}} \\
 = \frac{(4)(3 \times 3)}{(12)(10)} \\
 = \frac{36}{120} \\
 = \frac{3}{10} \\
 = 30\%
 \end{array}$$

The amount of cardboard wasted is 30%. (2 marks)

3. Use the Pythagorean Theorem to find the missing side.

$$\begin{array}{l}
 (BC)^2 + 5^2 = 13^2 \\
 (BC)^2 + 25 = 169 \\
 (BC)^2 + 25 - 25 = 169 - 25 \\
 (BC)^2 = 144 \\
 BC = \sqrt{144} \\
 = 12
 \end{array}$$

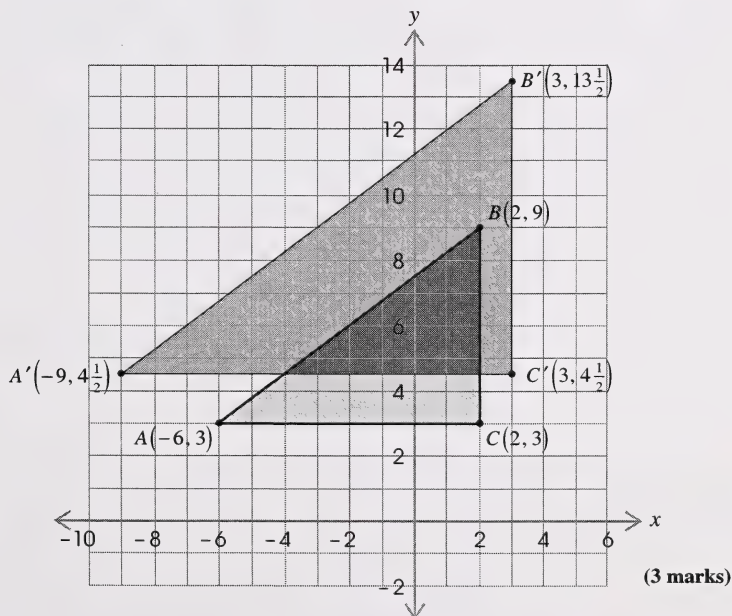
$$\begin{array}{l}
 \therefore \tan A = \frac{\text{opp}}{\text{adj}} \\
 = \frac{12}{5} \text{ or } 2\frac{2}{5} \quad (3 \text{ marks})
 \end{array}$$

4. Because the area of $\triangle GHI$ is 4 times the area of $\triangle JKL$, the length of each side of $\triangle GHI$ is $\sqrt{4} = 2$ times the length of each corresponding side of $\triangle JKL$. Therefore, because JL and GI are corresponding sides of similar triangles, the length of GI is $2 \times 6 = 12$ cm. (3 marks)

$$\begin{aligned} 5. \quad a. \quad A(-6, 3) &\rightarrow A' \left[\frac{3}{\cancel{2}} \left(\frac{-3}{\cancel{2}} \right), \frac{3}{2}(3) \right] = A' \left(-9, \frac{9}{2} \right) \\ &= A' \left(-9, 4\frac{1}{2} \right) \end{aligned}$$

$$\begin{aligned} B(2, 9) &\rightarrow B' \left[\frac{3}{\cancel{2}} (\cancel{2}), \frac{3}{2}(9) \right] = B' \left(3, \frac{27}{2} \right) \\ &= B' \left(3, 13\frac{1}{2} \right) \end{aligned}$$

$$\begin{aligned} C(2, 3) &\rightarrow C' \left[\frac{3}{\cancel{2}} (\cancel{2}), \frac{3}{2}(3) \right] = C' \left(3, \frac{9}{2} \right) \\ &= C' \left(3, 4\frac{1}{2} \right) \end{aligned}$$

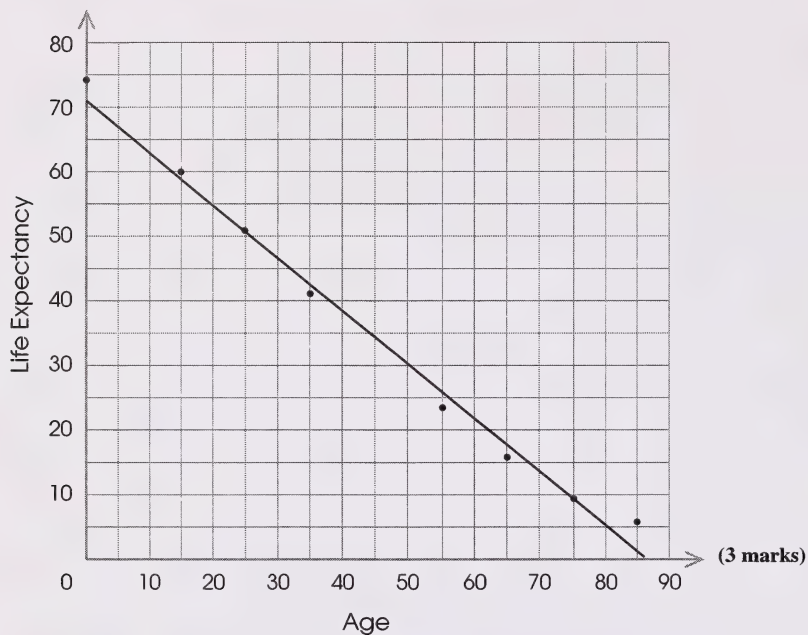


- b. Because all the sides are in proportion to each other, $\triangle ABC$ is similar to $\triangle A'B'C'$. (2 marks)

Part D: Statistics (11 marks)

1. a. Age is the independent variable. (1 mark)

b. **Life Expectancy of Males**



- c. As age increases, the number of years of life expectancy decreases. (1 mark)
- d. The life expectancy in the 1900s should be less than that of 1991. One reason is the advance in medicine since 1900. (2 marks)
2. Answers may vary. Sample answers are given.
- a. Yes, the bivariate data are related. As the number of kilometres increases, the cost of gas and food increases.
The points on the graph would rise to the right.
(2 marks)
- b. Yes, the bivariate data are related. As the temperature rises, the cost to heat homes decreases.
The points on the graph would fall to the right.
(2 marks)

MATHEMATICS PREPARATION 10

FINAL TEST

GENERAL INSTRUCTIONS

YOU HAVE **THREE** HOURS TO COMPLETE THIS TEST. Work through the entire test answering the questions you are sure you know. You will then be able to concentrate on the questions of which you are not quite sure.

TOTAL MARKS: 100

PART A: Numbers 40 marks

PART B: Patterns and Relations 26 marks

PART C: Shape and Space 23 marks

PART D: Statistics 11 marks

Note: You will need a scientific calculator and a geometry set.

Value

PART A: NUMBERS

40 Answer each of the following questions using the space provided.

1. Three families buy convenience food to save time. The Smith family buys convenience food every 2 days, the Ray family every 6 days, and the Taylor family every 10 days.

(2 marks)

- a. If May 31 was the last day all three families bought convenience food and all three families continue to follow their schedules, what is the next date when all three families buy convenience food?

(2 marks)

- b. How many times will only the Smith family and the Ray family buy convenience food on the same day, before all three families buy convenience food?

(3 marks)

2. The area of a rectangular blackboard is $4\frac{7}{8} \text{ m}^2$. If the blackboard is $1\frac{1}{2} \text{ m}$ wide, determine the length of the blackboard. Express your answer as a mixed fraction.

Name of Student _____

Student I.D. # _____

Name of School _____

Date _____

3. When the body is at rest, the number of heartbeats in one minute is known as the resting rate. The resting rate of a trained runner is 40 beats/min, whereas the resting rate of an unfit adult is 90 beats/min.

(2 marks)

- a. How many times faster is an unfit adult's heart rate compared to a trained runner's heart rate?

(1 mark)

- b. Express the answer to question 3.a. as a percent.

(3 marks)

4. Estimate the value of $\sqrt{60}$ to one decimal place. How could you check your estimate without using the $\sqrt{\quad}$ on your calculator?

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(3 marks)

5. A student did not have a protractor but needed to determine whether $\triangle ABC$ was a right triangle. He measured the sides and found they were 2 cm, 2.5 cm, and 1.5 cm. Is $\triangle ABC$ a right triangle?

(1 mark)

6. Write $(-4)^3$ in expanded form.

(2 marks)

7. Charlie thinks 3^{-4} is equal to $(-3)^4$. Is Charlie correct? Explain.

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8. Simplify the following expressions. Leave only positive exponents in your answers.

(2 marks)

a. $\frac{5x^2y^3}{x^4y^2}$

(2 marks)

b. $(-4x^2y^{-2})(2x)$

(2 marks)

c. $\frac{(2xy)(3x)}{(15xy)^0}$

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(2 marks)

9. The temperature dropped from -6°C to -15°C overnight. How much did the temperature change? Which integer represents this change in temperature?

(2 marks)

10. Sara modelled $+3 - (+5)$. She began by laying out three positive counters ($\oplus \oplus \oplus$) and then added two negative counters and two positive counters ($\ominus \ominus \oplus \oplus$) to her counters. Explain why Sara added negative counters.

11. Perform the following operations without a calculator.

(2 marks)

a. $\frac{2}{3} + \frac{1}{6} \div 1\frac{1}{2} - \frac{2}{9}$

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(2 marks)

b. $5 - \left[\left(\frac{1}{12} + \frac{2}{3} \right) + \frac{1}{4} \right]$

(2 marks)

c. $3^{-2} \times 3^4 + \left[2^2 + \left(\frac{1}{2} \right)^{-3} \right]$

12. A calculator was used to divide two numbers. The following appeared on the display.



(1 mark)

- a. What could the two numbers be?

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(1 mark)**b.** Are these the only two numbers that would give this result? Explain.

(3 marks)

- 13.** George buys Chinese food for his three friends and himself. He orders 2 meat dishes at \$7.50 each, 3 seafood dishes at \$6.20 each, 4 bowls of rice at \$0.75 each, and 4 bottles of pop at \$1.50 each. If George has \$50.00 and plans to leave a tip that is exactly 15%, how much money will he have left?

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Value**PART B: PATTERNS AND RELATIONS****26**

Answer each of the following questions using the space provided.

1. To order yearbooks, it costs \$1250 plus \$7 per yearbook ordered. A school hopes to order as many as possible to bring the price per yearbook down.

(3 marks)

- a. What would be the price per yearbook, if the school ordered 250 yearbooks?

(2 marks)

- b. How many yearbooks must the school order to bring the price down to \$9.50 per yearbook?

Name of Student _____

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Name of School _____

Date _____

(1 mark)

2. The cost of a large pizza is \$2.50 less than twice the cost of a small pizza. Write an expression for the cost of a large pizza. Identify your variables.

3. Simplify the following expressions.

(2 marks)

a. $(3a^3 - 2a^2 + 4) - (3a^3 - a^2 - 6)$

(2 marks)

b. $\frac{16n^4 - 8n^2 + 4n}{4n}$

(2 marks)

c. $(x+5)(x-2)$

Name of Student _____

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(5 marks)

4. Write out the steps explaining how you can use counters to model the algebraic solution for $2x + 7 = -x + 4$. Use diagrams in your explanation.

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Date _____

5. Factor each of the following.

(1 mark)

a. $3x^2y^2 - 6x$

(1 mark)

b. $x^2 - 4x - 12$

(3 marks)

6. Judy spent \$60 on clothes, which is 80% of the money she made baby-sitting. How much did Judy make baby-sitting?

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(3 marks)7. Solve $3x + 5 = -x + 41$, and check your solution.**(1 mark)**8. The volume of a cylinder is $V = \pi r^2 h$. Solve for the variable h .

Name of Student _____	Student I.D. # _____
Name of School _____	Date _____

Value**PART C: SHAPE AND SPACE****23** Answer each of the following questions using the space provided.**(5 marks)**

1. A cylindrical can of tomato juice is 18 cm high and has a diameter of 10.6 cm. Calculate the surface area of this cylinder. Round your answer to the nearest whole number.

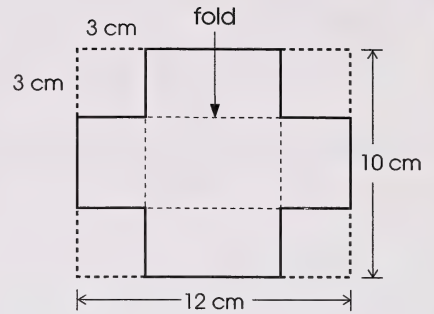
Name of Student _____

Student I.D. # _____

Name of School _____

Date _____

2. Four squares, each with a side of 3 cm, are cut out from a rectangular piece of cardboard measuring $12\text{ cm} \times 10\text{ cm}$. The cardboard is folded on the dotted lines to make a box (as shown in the diagram).



(3 marks)

- a. Find the length, width, and height of the box.

(2 marks)

- b. Calculate the volume of the box.

(2 marks)

- c. What percent of the cardboard is wasted? Round your answer to the nearest percent.

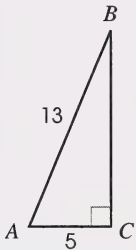
Name of Student _____

Student I.D. # _____

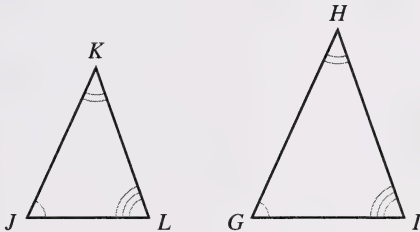
Name of School _____

Date _____

- (3 marks) 3. Use $\triangle ABC$ to determine $\tan A$. Leave your answer in fraction form.



- (3 marks) 4. Two similar triangles are given.

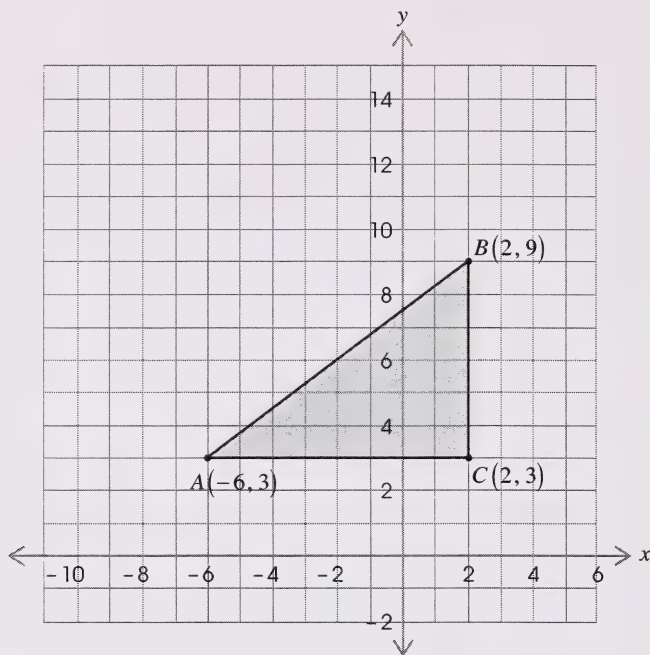


The area of $\triangle GHI$ is 4 times larger than the area of $\triangle JKL$. If JL is 6 cm in length, what is the length of GI ?

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(3 marks)

5. a. A mapping rule describes the transformation of $\triangle ABC$ to $\triangle A'B'C'$ as $(x, y) \rightarrow \left(\frac{3}{2}x, \frac{3}{2}y\right)$. Draw the $\triangle A'B'C'$.

**(2 marks)**

- b. Is $\triangle ABC$ similar to $\triangle A'B'C'$? Support your answer.

Name of Student _____

Student I.D. # _____

Name of School _____

Date _____

Value

PART D: STATISTICS

11

Answer each of the following questions using the space provided.

1. The following data was obtained to determine if the life expectancy of males is related to age. Use this information to answer the following questions. **Note:** In this case, life expectancy is the additional number of years a person should live.

Age	0	15	25	35	55	65	75	85
Life Expectancy	74.3	60.1	50.7	41.3	23.2	15.6	9.6	5.4

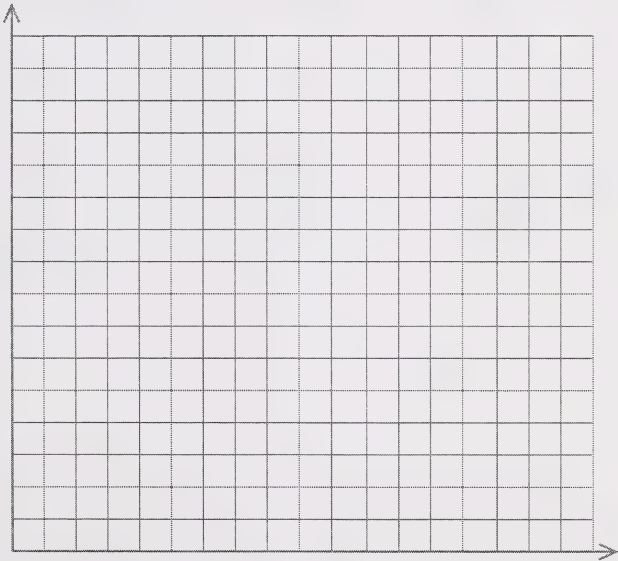
SOURCE: STATISTICS CANADA, HEALTH STATISTICS DIVISION

(1 mark)

- a. Which variable is the independent variable?

(3 marks)

- b. Make a scatter plot of the data on the following grid; then draw a line of best fit by inspection.



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(1 mark)

- c. Describe the apparent relationship between the variables in this data.

(2 marks)

- d. This data was collected in 1991. How do you think the life expectancy of a 0-year-old in 1991 would compare to the life expectancy of a 0-year-old in 1900. Explain.

2. Determine whether or not you believe the following bivariate data are related. If a scatter plot was drawn, what type of relationship would you predict and why.

(2 marks)

- a. distance travelled on a trip and cost of gas and food

(2 marks)

- b. daily temperature and cost to heat a house

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Name of School _____

Date _____

TEACHER QUESTIONNAIRE FOR MATHEMATICS PREPARATION 10

(© 2000)

This course is designed in a new distance learning format, so we are interested in your responses. Your constructive comments will be greatly appreciated, as future course revisions can then incorporate any necessary improvements.

Teacher's Name _____ Area of Expertise _____

School Name _____ Date _____

Design

1. The modules follow a definite systematic design. Did you find it easy to follow?

☐ Yes ☐ No If no, explain.

2. Did your observations reveal that the students found the design easy to follow?

☐ Yes ☐ No If no, explain.

3. Did you find the Teacher's Guide helpful?

☐ Yes ☐ No If no, explain.

4. Part of the design involves stating the objectives in student terms. Did you find this helped the students understand what they were going to learn?

☐ Yes ☐ No If no, explain.

5. The Teacher's Guide contains answers to the questions in the Assignment Booklet and may include a sample test. Did you find these helpful?

☐ Yes ☐ No If no, explain.

6. Did the Follow-up Activities prove to be helpful?

☐ Yes ☐ No If no, explain.

7. Were students motivated to try these Follow-up Activities?

☐ Yes ☐ No If no, give details.

8. Suggestions for computer and video activities may be included in the course. Were your students able to use these activities?

☐ Yes ☐ No Comment on the lines below.

9. Were the assignments appropriate?

☐ Yes ☐ No If no, give details.

10. Did you fax assignments? ☐ Yes ☐ No

11. If you did fax, did you get satisfactory results from using this procedure?

☐ Yes ☐ No If no, give details.

Instruction

1. Did you find the instruction clear?

☐ Yes ☐ No If no, give details.

2. Did your observations reveal that the students found the instruction interesting?

☐ Yes ☐ No If no, give details.

3. Did you find the instruction adequate?

☐ Yes ☐ No If no, give details.

4. Was the reading level appropriate?

☐ Yes ☐ No If no, give details.

5. Was the workload adequate?

☐ Yes ☐ No If no, give details.

6. Was the content accurate and current?

☐ Yes ☐ No If no, give details.

7. Did the content flow consistently and logically?

☐ Yes ☐ No If no, give details.

8. Was the transition between booklets smooth?

☐ Yes ☐ No If no, give details.

9. Was the transition between print and media smooth?

☐ Yes ☐ No If no, give details.

Additional Comments

Thanks for taking the time to complete this questionnaire.
Your feedback is important to us. Please return this
questionnaire to the address on the right.

Fax Number: (780) 674-6561

Instructional Design and Development
Learning Technologies Branch
Box 4000
Barrhead, Alberta
T7N 1P4

Note: Please ensure that each of your students has completed and forwarded a copy of the Course Survey.





Mathematics
Preparation 10
Teacher's Guide
Modules 1-5
2000